

Optimal Offline and Competitive Online Strategies for Transmitter-Receiver Energy Harvesting

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Abstract

A joint transmitter-receiver energy harvesting model is considered, where both the transmitter and receiver are powered by (renewable) energy harvesting source. Given a fixed number of bits, the problem is to find the optimal transmission power profile at the transmitter and ON-OFF profile at the receiver to minimize the transmission time. With infinite capacity at both the transmitter and receiver, optimal offline and optimal online policies are derived. The optimal online policy is shown to be two-competitive in the arbitrary input case. With finite battery capacities at both ends, only random energy arrival sequence with given distribution are considered, for which an online policy with bounded expected competitive ratio is proposed.

Index Terms

Energy harvesting, offline algorithm, online algorithm, competitive ratio.

I. INTRODUCTION

Extracting energy from nature to power communication devices has been an emerging area of research. Starting with [1], [2], a lot of work has been reported on finding the capacity, approximate capacity [3], structure of optimal policies [4], optimal power transmission profile [5]–[8], competitive online algorithms [9], etc. One thing that is common to almost all the prior work is the assumption that energy is harvested only at the transmitter while the receiver has some conventional power source. This is clearly a limitation, however, helped to get some critical insights into the problem.

In this paper, we broaden the horizon, and study the more general problem when energy harvesting is employed both at the transmitter and the receiver. The joint (tx-rx) energy harvesting model has not been studied in detail and only some preliminary results are available, e.g., a constant approximation to the maximum throughput has been derived in [10] or [11], [12]. This problem is fundamentally different than using energy harvesting only at the transmitter, where receiver is always assumed to have energy to stay *on*. In contrast to the variable power model at the transmitter where it can choose to transmit any power level given the available energy constraint, the receiver energy consumption model is binary, as it uses a fixed amount of energy to stay *on*, and is *off* otherwise. Since useful transmission happens only when the receiver is *on*, the problem is to find jointly optimal decisions about transmit power and receiver ON-OFF schedule. Under this model, there is an issue of coordination between the transmitter and the receiver to implement the joint decisions, however, we ignore that in the interest to make some analytical progress, and assume that the decisions are made by a centralized controller.

We study the canonical problem of finding the optimal transmission power and receiver ON-OFF schedule to minimize the time required for transmitting a fixed number of bits, first in the case when there is no limit on the battery capacities and then generalize it for finite battery capacities at both the transmitter and the receiver. We first consider the offline case, where the energy arrivals both at the transmitter and the receiver are assumed to be known non-causally. Even though offline scenario is unrealistic, it still gives some design insights. Then we consider the more useful online scenario, where both the transmitter and the receiver only have causal information about the energy arrivals. To characterize the performance of an online algorithm, typically, the metric of competitive ratio is used that is defined as the maximum ratio of ‘profit’ of the online and the offline algorithm over all possible inputs.

For the infinite battery capacity case, in prior work [5], an optimal offline algorithm has been derived for the case when energy is harvested only at the transmitter, which cannot be generalized with energy harvesting at the receiver together with the transmitter. To understand the difficulty, assume that the receiver can be *on* for maximum time T . The policy of [5] starts transmission at the first energy arrival time, and power transmission profile is the one that yields the tightest piecewise linear energy consumption curve that lies under the energy harvesting curve at all times and touches the energy harvesting curve at end time. The policy of [5], however, may take more than T time and hence may not be feasible with the receiver *on* time constraint. So, we may have to either delay the start of transmission and/or keep stopping in-between to accumulate more energy to transmit with higher power for shorter bursts, such that the total time for which transmitter and receiver is *on*, is less than T . Similarly, for the finite battery capacity, an optimal offline algorithm has been derived for the case when energy is harvested only at the transmitter in [13]. However, once again there is no easy way of extending the results of [13], when both the transmitter and receiver are powered by EH, and we need a new approach.

With infinite battery capacity at both the transmitter and the receiver, in the offline scenario, we derive the structure of the optimal algorithm, and then propose an algorithm that is shown to satisfy the optimal structure. The power profile of the

proposed algorithm is fundamentally different than the optimal offline algorithm of [5], however, the two algorithms have some common structural properties. The recipe of our solution is to first solve the simpler problem of finding the optimal offline algorithm when there is only one energy arrival at the receiver. Building upon this solution, we then derive the optimal offline solution to the problem with multiple energy arrivals at the receiver, to be one among finitely many solutions of the problem with only one energy arrival at the receiver, where corresponding single energy arrivals are suitably constructed. This technique not only gives an elegant method to prove the optimality, but also helps in simplifying the complexity of the optimal algorithm.

Next, we consider the more useful setup of online algorithms that use only causal information. With infinite battery capacities at both ends, for the online scenario, we propose an online algorithm, which starts at time where the accumulated energy at both the transmitter and the receiver is sufficient to transmit the given number of bits eventually. The transmit power at any time (only updated at energy arrival epoch of the transmitter) is such that using the available energy, the remaining number bits are transmitted in minimum time assuming no more energy is going to arrive in future. We show that the competitive ratio of the proposed online algorithm is strictly less than 2 for any energy arrival inputs, even if chosen by an adversary. With only energy harvesting at the transmitter, a 2-competitive online algorithm has been derived in [9]. This result is more general with different proof technique that allows energy harvesting at the receiver. To prove that the proposed online algorithm is optimal, we show a lower bound on the competitive ratio that is arbitrarily close to 2 for any online algorithm. This is accomplished by constructing two “bad” sequences of energy arrivals at the transmitter and the receiver, for which any algorithm fails to achieve a competitive ratio of better than 2 for at least one of the two sequences.

Finally, we consider the case of finite battery capacity. With finite battery capacity, it is easy to show that the competitive ratio of any online algorithm with the worst case input is unbounded as follows. Suppose, by time slot t , any online algorithm consumes more (less) energy than the optimal offline algorithm, then it is easy to construct future energy arrival sequences, for which the optimal offline algorithm can finish transmission of given number of bits, on account of knowing the input sequence and transmitting at a slower (faster) rate, while the online algorithm can never finish the transmission. Thus, we restrict ourselves to scenario where energy arrivals follow a known distribution, but the realization information is only known causally. We propose a simple Accumulate and Dump algorithm, that waits for battery to fill up to a certain prefixed level, and as soon as the accumulated energy is above the level, uses all the energy in the next slot, and restarts accumulating all over again. We show that the expected competitive ratio of the proposed algorithm is finite, which can be computed explicitly given the energy arrival distribution. In prior work [13]–[15], optimal offline algorithm has been derived when only the transmitter is powered with EH and has a finite battery capacity. Instead of the offline regime, in this paper, we concentrate on the online setting which is more relevant in practice and propose algorithms that have a finite penalty with respect to the optimal offline algorithm.

II. SYSTEM MODEL

The energy arrival instants at transmitter are marked by τ_i 's with energy \mathcal{E}_i 's for $i \in \{0, 1, \dots\}$. The total energy harvested at the transmitter till time t is given by

$$\mathcal{E}(t) = \sum_{i: \tau_i \leq t} \mathcal{E}_i. \quad (1)$$

Similarly, the energy arrival instants at the receiver are denoted as r_i with energy \mathcal{R}_i . We initialize τ_0, r_0 to 0 without affecting the system model as follows. If $r_0 \leq \tau_0$, i.e. the first energy arrival at the receiver occurs before the first energy arrival at the transmitter, then we assume that $\sum_{i: r_i \leq \tau_0} \mathcal{R}_i$ energy is harvested at the receiver at time τ_0 , i.e. $r_0 = \tau_0$. We shift the time origin to $\tau_0 = r_0$, i.e. $\tau_0 = r_0 = 0$. Note that, since the transmitter has 0 energy to transmit before time τ_0 , no transmission policy can start transmission before τ_0 . Therefore, assuming $r_0 = \tau_0$ whenever $r_0 \leq \tau_0$, does not affect any transmission policy. Similarly, whenever $\tau_0 < r_0$, we assume $\sum_{i: \tau_i \leq r_0} \mathcal{E}_i$ energy arrives at the transmitter at time r_0 , i.e. $\tau_0 = r_0$, and we offset time origin to $\tau_0 = r_0 = 0$.

The receiver spends a constant P_r amount of power to be in ‘on’ state during which it can receive data from the transmitter. When it is in ‘off’ state it does not receive data, and uses no power. Hence, each energy arrival of \mathcal{R}_i adds $\Gamma_i = \frac{\mathcal{R}_i}{P_r}$ amount of receiver on time. The total ‘time’ harvested at the receiver till time t is given by,

$$\Gamma(t) = \sum_{i: r_i \leq t} \Gamma_i. \quad (2)$$

The rate of transmission using transmit power p when the receiver is on is given by a function $g(p)$ which is assumed to follow the following properties,

- P1) $g(p)$ is monotonically increasing in p , such that $g(0) = 0$ and $\lim_{p \rightarrow \infty} g(p) = \infty$,
- P2) $g(p)$ is concave in nature with p ,
- P3) $\frac{g(p)}{p}$ is convex, monotonically decreasing with p and $\lim_{p \rightarrow \infty} \frac{g(p)}{p} = 0$.

Assuming an AWGN channel, log function is one such example satisfying all the above properties.

Let a transmission policy change its transmission power at time instants s_i 's, i.e. p_i is the transmitter power between time s_i and s_{i+1} . The receiver is *on* from time s_i to s_{i+1} whenever $p_i \neq 0$ and is *off* only if $p_i = 0$. Thus, succinctly, we say that receiver is *on* at time t to mean that transmit power $p_i \neq 0$ for $t \in [s_i, s_{i+1}]$ and receiver is *on*. The start and the end time of any policy is denoted by s_1 and s_{N+1} , respectively. Thus, any policy can be represented as $\{\mathbf{p}, \mathbf{s}, N\}$, where $\mathbf{p} = \{p_1, p_2, \dots, p_N\}$ and $\mathbf{s} = \{s_1, s_2, \dots, s_{N+1}\}$. The energy used by a policy at the transmitter upto time t is denoted by $U(t)$, and the number of bits sent by time t is represented by $B(t)$. Clearly, for $j = \arg \max_i \{s_i < t\}$,

$$U(t) = \sum_{i=1, p_i \neq 0}^{j-1} p_i(s_{i+1} - s_i) + p_j(t - s_j), \quad s_1 < t \leq s_{N+1}, \quad (3)$$

$$= U(s_{N+1}), \quad t > s_{N+1}, \quad (4)$$

$$= 0, \quad t \leq s_1, \quad (5)$$

$$B(t) = \sum_{i=1, p_i \neq 0}^{j-1} g(p_i)(s_{i+1} - s_i) + g(p_j)(t - s_j), \quad s_1 < t \leq s_{N+1}, \quad (6)$$

$$= B(s_{N+1}), \quad t > s_{N+1}, \quad (7)$$

$$= 0, \quad t \leq s_1. \quad (8)$$

Similarly, the total time for which the receiver is *on* till time t is denoted as $C(t)$.

Except for section VII, we assume that an infinite battery capacity is available both at the transmitter and the receiver to store the harvested energy. Our objective is, given a fixed number of bits B_0 , minimize the time of their transmission. For any policy, the total time for which the receiver is *on* is referred to as the 'transmission time' or the 'transmission duration', and the time by which the transmission of B_0 bits is finished, is called as the 'finish time'. Thus, we want to minimize the finish time. Also, since the receiver may not be always *on* before finish time, we have transmission time less than or equal to finish time. Formally, we want to solve,

$$\min_{\{\mathbf{p}, \mathbf{s}, N\}, T=s_{N+1}} T \quad (9)$$

$$\text{subject to} \quad B(T) = B_0, \quad (10)$$

$$U(t) \leq \mathcal{E}(t) \quad \forall t \in [0, T], \quad (11)$$

$$C(t) \leq \Gamma(t) \quad \forall t \in [0, T]. \quad (12)$$

Under transmission policy $\{\mathbf{p}, \mathbf{s}, N\}$, the total receiver *on* time till time t for $s_1 < t \leq s_{N+1}$ is given by,

$$C(t) = \sum_{i=1}^{k-1} \mathbb{1}_i(s_{i+1} - s_i) + \mathbb{1}_k(t - s_k), \quad (13)$$

where $k = \max\{i | s_i < t\}$ and $\mathbb{1}_i : \mathbb{R} \rightarrow \{0, 1\}$ is a function that takes value 1 if $p_i > 0$ and 0 if $p_i = 0$. Constraints (11) and (12) are the energy neutrality constraints at the transmitter and the receiver, i.e. energy/on-time used cannot be more than available energy/on-time

III. OPTIMAL OFFLINE ALGORITHM FOR SINGLE ENERGY ARRIVAL AT THE RECEIVER

In this section, we consider an offline scenario, i.e., all energy arrival epochs τ_i 's and energy harvest amounts \mathcal{E}_i 's at the transmitter are known ahead of time non-causally. Moreover, we assume that the receiver gets only one energy arrival of \mathcal{R}_0 at time 0, and hence the total receiver *on* time is $\Gamma_0 = \frac{\mathcal{R}_0}{P_r}$. The crux of problem in both cases (with single/multiple energy arrivals at the receiver) lies in overcoming the problem of the limited transmission time available at the receiver and is not affected much by the number of energy harvests at the receiver. As we shall see, the optimal offline algorithm with multiple energy arrivals at the receiver (solving (9)) consists of repeated application of the derived optimal algorithm for the single energy arrival case. Hence, we postpone the analysis with multiple energy arrivals at the receiver to section V.

With only one energy harvest at the receiver, i.e. with total receiver time Γ_0 harvested at time 0, a special case of (9) to minimize the finish time of transmission of B_0 bits is,

$$\min_{\{p, s, N\}, T=s_{N+1}} T \quad (14)$$

$$\text{subject to } B(T) = B_0, \quad (15)$$

$$U(t) \leq \mathcal{E}(t), \quad \forall t \in [0, T], \quad (16)$$

$$\sum_{i=1: p_i \neq 0}^N (s_{i+1} - s_i) \leq \Gamma_0. \quad (17)$$

Compared to the no receiver constraint [5], Problem (14) is far more complicated, since it involves jointly solving for optimal transmitter power allocation and time for which to keep the receiver *on*.

We next present some structural results on the optimal policy to (14) starting with Lemma 1, which states that transmission powers in the optimal policy to (14) are non-decreasing over time.

Lemma 1. *In an optimal solution to Problem (14), if $p_i \neq 0$, then $p_i \geq p_j \quad \forall j < i$ with $i, j \in \{1, 2, \dots, N\}$ ¹.*

Proof. We prove this by contradiction. Assume that the optimal policy (say X), with $\{p, s, N\}$ violates the condition stated in Lemma 1. Let $p_i \neq 0$ be the first transmission power such that $\exists k < i : p_i < p_k$. Let $j = \max\{k : p_i < p_k\}$.

Case 1 : Suppose $j = i - 1$. This situation is shown in Fig. 1 (a). In this case, consider a new transmission policy (say Y) which is same as the optimal policy till time s_{i-1} . From s_{i-1} to s_{i+1} , Y transmits at a constant power $p' = \frac{p_i(s_{i+1} - s_i) + p_{i-1}(s_i - s_{i-1})}{s_{i+1} - s_{i-1}}$. Then the number of bits transmitted by policy Y from time s_{i-1} to s_{i+1} is given by $g(p')(s_{i+1} - s_{i-1})$ while the optimal policy transmits $g(p_i)(s_{i+1} - s_i) + g(p_{i-1})(s_i - s_{i-1})$ bits. Due to concavity of $g(p)$,

$$\begin{aligned} & g(p_i) \frac{s_{i+1} - s_i}{s_{i+1} - s_{i-1}} + g(p_{i-1}) \frac{s_i - s_{i-1}}{s_{i+1} - s_{i-1}} \\ & \leq g\left(\frac{p_i(s_i - s_{i-1}) + p_{i-1}(s_{i+1} - s_i)}{s_{i+1} - s_{i-1}}\right), \\ & g(p_i)(s_{i+1} - s_i) + g(p_{i-1})(s_i - s_{i-1}) \\ & \leq g(p')(s_{i+1} - s_{i-1}). \end{aligned}$$

Hence, both X and Y transmit equal number of bits till time s_{i-1} , while Y transmits more number of bits than X by time s_{i+1} . After time s_{i+1} , suppose policy Y transmits with power same as policy X till it completes transmitting B_0 bits. Since Y has transmitted more bits than X till time s_{i+1} , it finishes transmitting all B_0 bits earlier than X , contradicting the optimality of X .

Case 2 : When $j < i - 1$, by our assumption on choosing j , $p_i > p_{j+1}, \dots, p_{i-1}$ and $p_i < p_j$. So, $p_{i-1}, \dots, p_{j+1} < p_j$. If any of p_{i-1}, \dots, p_{j+1} is non zero, then i no longer remains the minimum index violating the condition stated in Lemma 1. Hence, $p_{i-1}, \dots, p_{j+1} = 0$. This situation is shown in Fig. 1(b). Now, consider a policy W where the transmission power is same as the optimal policy before time s_j and after time s_{i+1} . From s_j to $s'_j = s_j + s_i - s_{j+1}$, W keeps the receiver *off* (so transmitter does not transmit in this duration) and from s'_j to s_i it transmits at power p_j . This policy still transmits equal number of bits and ends at the same time as the optimal policy X . Now that W matches with the form of X in *Case 1* from time s'_j to s_{i+1} , we could proceed to generate another policy form W (like Y in *Case 1*) which would finish earlier than W . Hence, this new policy would finish earlier than X as well and we would reach a contradiction. \square

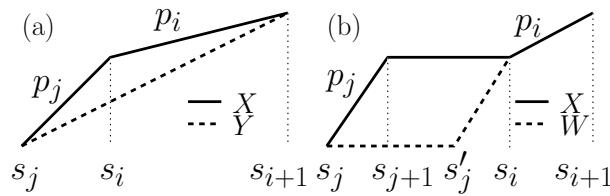


Fig. 1. Figure showing the two cases of Lemma 1, (a)Case 1 and (b)Case 2, with $p_i > p_j$.

Although, Lemma 1 is valid for every optimal policy to (14), we will narrow down the search for optimal solutions by looking at an interesting property presented in Lemma 2, which tells us that there is no need to stop in-between transmissions,

¹Observe that without receiver energy harvesting constraint (17), $p_i \neq 0, \forall i$ from [5] and Lemma 1 would be same as Lemma 1 in [5]. But, as we have constraint on the total receiver time, in optimal solution, transmitter may shut *off* for some time and resume transmission when enough energy is harvested. Hence, p_i may be 0 in-between transmission. Lemma 1 shows that even if this happens, non-zero powers still remain non-decreasing.

and start again. Thus, without affecting optimality, the start of the transmission can be delayed so that transmission power is non-zero throughout.

Lemma 2. *The optimal solution to Problem (14) may not be unique, but there always exists an optimal solution where once the transmission has started, the receiver remains ‘on’ throughout, until the transmission is complete.*

Proof. We construct an optimal solution for which $p_i > 0$ for all $i \in \{1, \dots, N\}$, i.e., with no breaks in transmission, from any other optimal solution. Let an optimal policy X be characterized by $\{\mathbf{p}, \mathbf{s}, N\}$. Now, if $p_i \neq 0 \forall i$, then we are done. Suppose some powers, say $p_{i_1}, p_{i_2}, \dots, p_{i_k} = 0$ for some $k < N$, where $i_1 < i_2 < \dots < i_k$. We first look at instant i_1 .

Consider Fig. 2 (a), and a new policy (say Y) which is same as policy X before time s_{i_1-1} and after time s_{i_1+1} . But, it keeps the receiver *off* for a duration of $(s_{i_1+1} - s_{i_1})$ starting from time s_{i_1-1} (i.e. from s_{i_1-1} to $s'_{i_1} = (s_{i_1-1} + s_{i_1+1} - s_{i_1})$) and transmits with power p_{i_1-1} from time s'_{i_1} till s_{i_1+1} . Y transmits same amount of bits in same time as X and also satisfies constraints (15)-(17). So Y is also an optimal policy. But the receiver *off* duration in Y , $(s_{i_1+1} - s_{i_1})$, has been shifted to left.

Next, we generate another policy Z from Y by shifting the *off* duration $s'_{i_1} - s_{i_1-1} = (s_{i_1+1} - s_{i_1})$ to start from epoch s_{i_1-2} upto s'_{i_1-1} , $s'_{i_1-1} - s_{i_1-2} = s'_{i_1} - s_{i_1-1} = (s_{i_1+1} - s_{i_1})$, as shown Fig. 2 (b). p_{i_1-2} is shifted right to start from s'_{i_1-1} . Note that Z is also optimal. We continue this process of shifting the receiver *off* period to the left to generate new optimal policies till we reach a policy (say W) where the receiver is *off* for time $(s_{i_1+1} - s_{i_1})$ from s_1 , i.e. from s_1 to s'_1 , $s'_1 - s_1 = (s_{i_1+1} - s_{i_1})$, as shown in Fig. 2 (c). As W has 0 transmission power from the start time s_1 to s'_1 , the effective start time of W can now be changed to s'_1 .

We can repeat this procedure for each *off* period corresponding to p_{i_2}, \dots, p_{i_k} till the total *off* period is shifted to the beginning of transmission. This results in a policy with no zero powers in between, that starts *after* time s_1 (at $s_1 + (s_{i_1+1} - s_{i_1}) + \dots + (s_{i_k+1} - s_{i_k})$) and ends at the same time s_{N+1} as policy X . □

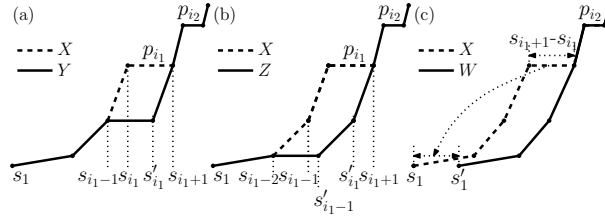


Fig. 2. Illustration of Lemma 2. Receiver *off* time of $(s_j - s_{i_1})$ is progressively shifted to left as shown in (a) to (b) to (c).

In the subsequent discussion, the optimal solution to Problem (14) means one with no breaks in transmission (reception). As we shall see in Theorem 1, such an optimal solution is unique.

Next, we show that the transmission power changes (if at all) only at energy arrival epochs τ_i 's, and the energy used up by that epoch is equal to all the energy that has arrived till then.

Lemma 3. *For optimal policy $\{\mathbf{p}, \mathbf{s}, N\}$, $s_i = \tau_j$ for some j , $U(s_i) = \mathcal{E}(s_i^-) \forall i \in \{2, \dots, N\}$, and $U(s_{N+1}) = \mathcal{E}(s_{N+1}^-)$.*

Proof. By Lemma 1 and 2, $p_i \neq 0$ and $p_{i+1} \geq p_i, \forall 1 \leq i \leq N$. So, the proof follows similar to Lemma 2,3 in [5]. □

It may happen that at some epoch τ_k , $U(\tau_k) = \mathcal{E}(\tau_k^-)$ holds true, but the transmission power does not change. For notational simplicity, we include all such τ_k 's in \mathbf{s} , where $U(\tau_k) = \mathcal{E}(\tau_k^-)$.

Next lemma states that if we take any feasible policy, $\{\mathbf{p}, \mathbf{s}, N\}$ and decrease p_1 and increase p_N while keeping the number of transmitted bits fixed, the transmission time increases, while reducing the finish time of the policy. Lemma 4 will be useful to prove uniqueness of the optimal policy with no breaks in transmission.

Lemma 4. *Consider two policies X , $\{\mathbf{p}, \mathbf{s}, N\}$ and Y , $\{\tilde{\mathbf{p}}, \tilde{\mathbf{s}}, N\}$, which are feasible with respect to energy constraint (16), have non-decreasing powers and transmit same number of bits in total. If Y is same as X from time s_2 to s_N , but $\tilde{p}_1 = p_1 - \alpha, \tilde{p}_N = p_N + \beta$ with $\alpha, \beta > 0$ and $U(s_{N+1}) = U(\tilde{s}_{N+1})$, then we have that the finish time with Y is less than that of X , i.e., $\tilde{s}_1 = s_1 - \gamma, \tilde{s}_{N+1} = s_{N+1} - \delta$ with some $\gamma, \delta > 0$, and the transmission time of Y is more than that of X , i.e., $(\tilde{s}_{N+1} - \tilde{s}_1) > (s_{N+1} - s_1)$.*

Proof. X and Y having used same amount of energy from s_2 to s_{N+1} , we can say that $\mathcal{E}(\tilde{s}_2) - \mathcal{E}(\tilde{s}_1) = \mathcal{E}(s_2) - \mathcal{E}(s_1)$, and $\mathcal{E}(\tilde{s}_{N+1}) - \mathcal{E}(\tilde{s}_N) = \mathcal{E}(s_{N+1}) - \mathcal{E}(s_N)$. Thus, we can define $\gamma = \frac{\alpha}{p_1 - \alpha}(s_2 - s_1)$ and $\delta = \frac{\beta}{p_N + \beta}(s_{N+1} - s_N)$. As X and Y transmit equal number of bits in total and are identical between time s_2 and s_N , we can just equate the number of bits transmitted by X before s_1 and after s_N (LHS of (18)) with that of Y (RHS of (18)), i.e.,

$$\begin{aligned} g(p_N)(s_{N+1} - s_N) + g(p_1)(s_2 - s_1) \\ = g(\tilde{p}_N)(\tilde{s}_{N+1} - \tilde{s}_N) + g(\tilde{p}_1)(\tilde{s}_2 - \tilde{s}_1), \end{aligned} \quad (18)$$

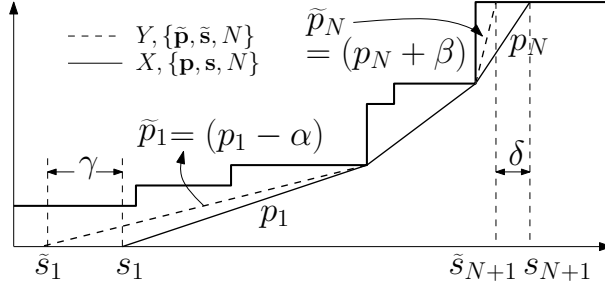


Fig. 3. Illustration for the proof of Lemma 4.

(Note that only one of the four variable $\alpha, \beta, \gamma, \delta$ can be independently chosen.) Therefore, from (18),

$$\begin{aligned} & (p_N + \beta)p_N \frac{\delta}{\beta} \left(\frac{g(p_N)}{p_N} - \frac{g(p_N + \beta)}{p_N + \beta} \right) \\ &= (p_1 - \alpha)p_1 \frac{\gamma}{\alpha} \left(\frac{g(p_1 - \alpha)}{p_1 - \alpha} - \frac{g(p_1)}{p_1} \right). \end{aligned} \quad (19)$$

As $g(p)/p$ is a continuous & differentiable function, the mean value theorem implies that $\exists p'_N : p_N < p'_N < p_N + \beta$ and $p'_1 : p_1 - \alpha < p'_1 < p_1$ such that

$$\left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_N} = \frac{1}{\beta} \left(\frac{g(p_N + \beta)}{p_N + \beta} - \frac{g(p_N)}{p_N} \right) \text{ and} \quad (20)$$

$$\left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_1} = -\frac{1}{\alpha} \left(\frac{g(p_1 - \alpha)}{p_1 - \alpha} - \frac{g(p_1)}{p_1} \right). \quad (21)$$

Substituting (20) and (21) in (19) we get,

$$\delta p_N p_N \left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_N} = \gamma p'_1 p_1 \left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_1}. \quad (22)$$

Now $\frac{d}{dp} \frac{g(p)}{p}$ is an increasing function of p since $g(p)/p$ is convex. Hence, with $p'_1 < p_1 \leq p_N < p'_N$,

$$\left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_N} > \left. \frac{d}{dp} \frac{g(p)}{p} \right|_{p=p'_1}. \quad (23)$$

Thus, (22) implies $\gamma > \delta$. So, transmission time in the policy Y , $(s_{N+1} - s_1 + \gamma - \delta)$, is greater than the transmission time in policy X i.e. $(s_{N+1} - s_1)$. \square

Lemma 5 uses Lemma 4 to prove that if the start time of the optimal policy is delayed beyond the first 'time' arrival instant $r_0 = 0$ at the receiver, then the transmission time will be equal to Γ_0 , i.e., it will exhaust all the transmission time available with the receiver.

Lemma 5. For an optimal policy $\{p, s, N\}$, either $s_{N+1} - s_1 = \Gamma_0$ or $s_1 = r_0 = 0$.

Proof. We use contradiction to prove the result. Suppose the optimal policy say X , starts at $s_1 > 0$ and has transmission time $(s_{N+1} - s_1) < \Gamma_0$. We will generate another policy which has finish time less than that of X , having transmission time squeezed in between $(s_{N+1} - s_1)$ and Γ_0 . Consider policy Y ($\{\tilde{p}, \tilde{s}, N\}$) in relation to X , as defined in Lemma 4. As $\alpha, \beta, \delta, \gamma$ are all related (by constraints presented in Lemma 4), choice of one variable (we consider α) defines Y . By definition of s_i 's, s_2 is the first energy arrival which is on the boundary of energy constraint (16) i.e. $U(s_2) = \mathcal{E}(s_2^-)$ and s_N is the last epoch satisfying $U(s_N) = \mathcal{E}(s_N^-)$. Hence, we can choose $\alpha > 0$, such that \tilde{p}_1 and \tilde{p}_N would be feasible with respect to energy constraint (16). Note that if $s_1 = 0$, then any value of α would have made \tilde{p}_1 infeasible.

From Lemma 4, we know that the transmission time of policy Y is more than that of X , i.e. $(\tilde{s}_{N+1} - \tilde{s}_1) > (s_{N+1} - s_1)$. From the hypothesis $(s_{N+1} - s_1) < \Gamma_0$. Therefore, let $(s_{N+1} - s_1) = \Gamma_0 - \epsilon$, with $\epsilon > 0$. If the chosen value of α is such that $\gamma - \delta \leq \epsilon$, then $(\tilde{s}_{N+1} - \tilde{s}_1) < \Gamma_0$. If not, then we can further reduce α so that $\gamma - \delta \leq \epsilon$ ($\alpha, \beta, \gamma, \delta$ being related by continuous functions). Note that, when $\epsilon = 0$, any choice of α would make $(\tilde{s}_{N+1} - \tilde{s}_1) > \Gamma_0$. Hence, with this choice of α , $(s_{N+1} - s_1) < (\tilde{s}_{N+1} - \tilde{s}_1) < \Gamma_0$ holds and policy Y contradicts the optimality of policy X (as finish time of Y is less than finish time of X , $\tilde{s}_{N+1} = s_{N+1} - \delta < s_{N+1}$ from Lemma 4). Thus $s_{N+1} - s_1 = \Gamma_0$ if $s_1 \neq 0$ in an optimal policy. \square

Summarising the results of Lemmas 1-5, the optimal policy $\{p, s, N\}$ may change transmission powers only at energy arrival epochs i.e. $\forall i \in \{2, \dots, N\}$, $s_i = \tau_j$ for some j . At these epochs, it exhausts the total energy available i.e. $U(s_i) = \mathcal{E}(s_i^-)$.

The transmission powers are also non-decreasing with time, and the optimal policy uses up the total ‘receiver time’ allowed, if it does not start transmitting from $r_0 = 0$.

Now we prove in Theorem 1 that the structure described in Lemma 1-5 including Lemma 6 (for ease of presentation Lemma 6 is postponed to section IV) is not only necessary, but is indeed sufficient for optimality of a policy.

Theorem 1. A policy $\{p, s, N\}$ is an optimal solution to Problem (14) if and only if,

$$\sum_{i=1}^{i=N} g(p_i)(s_{i+1} - s_i) = B_0; \quad (24)$$

$$p_1 \leq p_2 \leq \dots \leq p_N; \quad (25)$$

$$s_i = \tau_j \quad \text{for some } j, i \in \{2, \dots, N\} \quad \text{and} \quad U(s_i) = \mathcal{E}(s_i^-), \quad \forall i \in \{2, \dots, N+1\}; \quad (26)$$

$$s_{N+1} - s_1 = \Gamma_0, \quad \text{if } s_1 > 0 \quad \text{or} \quad s_{N+1} \leq \Gamma_0, \quad \text{if } s_1 = 0; \quad (27)$$

$$\exists s_j : s_j \in s \quad \text{and} \quad s_j = \tau_q, \quad (28)$$

where τ_q is defined in *INIT_POLICY* of section IV.

Proof. The proof consists of establishing both necessary and sufficiency conditions. The necessity of (24) follows as it is a constraint to the Problem (14), (25) follows from Lemma 1, 2, (26) follows from Lemma 3, (27) follows from Lemma 5, and (28) follows from Lemma 6.

Now, we prove the sufficiency of the structure (24)-(28). Let a policy $X, \{p, s, N\}$ follow structure (24)-(28). We need to show that this policy is optimal, which we do via contradiction. Suppose X is not optimal. Let there exists another policy $Y, \{p', s', N'\}$ which is optimal. Since Y abides by Lemma 1-6 on account of its optimality, Y also satisfies structure (24)-(28). (Now both X and Y satisfy structure (24)-(28) but Y is optimal i.e. it finishes before X . This would mean that there possibly exists some more conditions which are followed by Y but not X). We need to show that such a optimal policy Y (different from X) cannot exist or is infeasible, i.e., both X and Y cannot simultaneously satisfy (24)-(28) and be different.²

The following cases arise depending on whether $s'_1 > s_1$, $s'_1 = s_1$ or $s'_1 < s_1$.

Case1: If $s'_1 > s_1 \geq 0$, then by (27), $s'_{N'+1} = s'_1 + \Gamma_0 > s_1 + \Gamma_0 \geq s_{N+1}$. So policy Y finishes after time s_{N+1} and hence cannot be optimal.

Case2: Suppose $s'_1 = s_1$. Let s'_i be the first epoch for which $p'_i \neq p_i$ for some $i \in \{1, 2, \dots, N\}$.

Suppose $p'_i > p_i$. If, in policy Y , transmission continues after s_{i+1} i.e. $s'_{N'+1} > s_{i+1}$, then the amount of energy used by Y in interval $[s_i, s_{i+1}]$ can be lower bounded by $p'_i(s_{i+1} - s_i)$, which follows from (25). Since $p'_i > p_i$, $p'_i(s_{i+1} - s_i)$ is more than $p_i(s_{i+1} - s_i)$, which is the energy used by policy X . But by structure (26), X uses all energy available at both s_i and s_{i+1} . So, the maximum energy available in $[s_i, s_{i+1}]$ is $p_i(s_{i+1} - s_i)$. Therefore, Y uses more than available energy in $[s_i, s_{i+1}]$ and is not feasible with respect to the energy constraint.

If $s'_{N'+1} \leq s_{i+1}$, then it can be easily verified by concavity of function $g(p)$ that Y transmits strictly less number of bits in interval $[s_i, s_{N'+1}]$ than X in interval $[s_i, s_{i+1}]$. Both policies being same till s_i , we conclude that Y transmits less than B_0 bits by its finish time $s_{N'+1}$, and thus it is not feasible with respect to (24).

When $p_i > p'_i$, symmetrical arguments follow.

Case3: This case argues the infeasibility of Y when $0 \leq s'_1 < s_1$. Since $s_1 > 0$, transmission time of X is equal to Γ_0 from (27). The idea of the proof is to show that if an optimal policy Y starts its transmission early and finishes earlier than policy X , it always takes more transmission time than X ($= \Gamma_0$), which is going to violate the time constraint (17). First, we establish that Y must be same as policy X from epoch s_2 to an epoch s_j such that $s_j = \max_{s_i < s'_{N'+1}} s_i$. Let $s'_k = \max_{s'_i < s_2} s'_i$, and

Y continue from s'_k with constant power p'_k till s'_{k+1} . Clearly $s'_{k+1} \geq s_2$ from definition of s'_k .

Suppose $s'_{k+1} > s_2$. Since transmission with a constant power p'_k from s'_k to s'_{k+1} is feasible, transmission with constant power $\frac{\mathcal{E}(s_2^-) - \mathcal{E}(s'_k)}{(s_2 - s'_k)}$ from s'_k to s_2 , and $\frac{\mathcal{E}(s'_{k+1}) - \mathcal{E}(s_2^-)}{(s'_{k+1} - s_2)}$ from s_2 to s'_{k+1} is also feasible for any policy (Refer to Fig. 4 (a)) and hence,

$$\frac{\mathcal{E}(s'_{k+1}) - \mathcal{E}(s_2^-)}{(s'_{k+1} - s_2)} < \frac{\mathcal{E}(s_2^-) - \mathcal{E}(s'_k)}{(s_2 - s'_k)}. \quad (29)$$

Transmission with power $\frac{\mathcal{E}(s_2^-) - \mathcal{E}(s'_k)}{(s_2 - s'_k)}$ exhausts all available energy at epochs s'_k and s_2 . Therefore, power $p_1 = \frac{\mathcal{E}(s_2^-)}{(s_2 - s_1)}$

(in policy X) from s_1 to s_2 must be greater than $\frac{\mathcal{E}(s_2^-) - \mathcal{E}(s'_k)}{(s_2 - s'_k)}$. If not, then transmission with power p_1 in X would become

²Note that Lemma 2 suggests that optimal solution to Problem (14) may not be unique in general, but Theorem 1 shows that the optimal solution *without breaks* in transmission is indeed unique.

infeasible. Thus, from (29),

$$\frac{\mathcal{E}(s'_{k+1}) - \mathcal{E}(s_2^-)}{(s'_{k+1} - s_2)} < p_1. \quad (30)$$

Now, transmission with power p_1 from s_1 to s_2 , and transmission with power $\frac{\mathcal{E}(s'_{k+1}) - \mathcal{E}(s_2^-)}{(s'_{k+1} - s_2)}$ from s_2 to s'_{k+1} are both feasible for any policy. This combined with (30) would imply transmission with a constant power $\frac{\mathcal{E}(s'_{k+1})}{(s'_{k+1} - s_1)}$ from s_1 to s'_{k+1} is feasible and hence,

$$\frac{\mathcal{E}(s'_{k+1})}{(s'_{k+1} - s_1)} < p_1. \quad (31)$$

Since finish time of X , $s_{N+1} = s_1 + \Gamma_0 > s'_1 + \Gamma_0 \geq s'_{N'+1} \geq s'_{k+1}$, X transmits in interval $[s_1, s'_{k+1}]$ and uses atleast $p_1(s'_{k+1} - s_1)$ energy in this interval, which follows from (25). But, the maximum energy available for transmission in interval $[s_1, s'_{k+1}]$ is $\mathcal{E}(s'_{k+1})$. From (31), we can infer that X uses more than this available energy in $[s_1, s'_{k+1}]$, and therefore, we reach a contradiction over feasibility of X . So, our hypothesis, $s'_{k+1} > s_2$, is incorrect. Since, $s'_{k+1} \geq s_2$, we can conclude that $s'_{k+1} = s_2$.

Now, let $p'_{k+1} \neq p_2$ and $s_j > s_3$. From the definition of p_2 , $p_{k+1} > p_2$. Then the amount of energy used by policy Y between s_2 and s_3 is more than what is available. So $p'_{k+1} = p_2$ ($s'_{k+2} = s_3$) and similarly, we can show that $p'_{k+2} = p_3 \cdots$ ($s'_{k+3} = s_4 \cdots$) till epoch s_j . This completes the proof that Y is same as policy X from epoch s_2 to s_j .

By structure (28) we can be sure that there exists atleast one epoch $s_i = \tau_q$ which belongs to \mathbf{s} as well as \mathbf{s}' . So, $j \geq 2$.

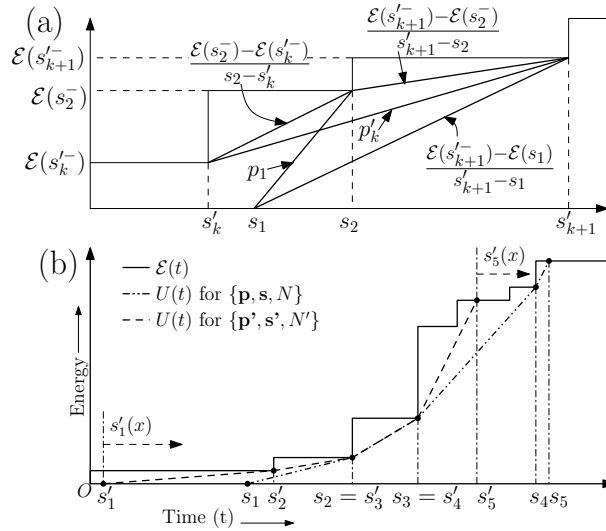


Fig. 4. Energy curves at transmitter explaining *Case3* in proof of Theorem 1

Continuing with *Case3*, consider the following process which creates feasible policies from policy $\{\mathbf{p}', \mathbf{s}', N'\}$ as shown in Fig. 4 (b). We define two pivots l and r . Initially we set $l = s'_2$ and $r = s'_{N'}$. The transmission power right before l is u ($u = p'_1$ initially) and right after r is v ($v = p'_{N'}$ initially). Keeping the policy $\{\mathbf{p}', \mathbf{s}', N'\}$ same from l to r , we increase u by a small amount to $u + du$ and decrease v by a small amount to $v - dv$ such that the number of bits transmitted (i.e. B_0) remains same under this transformation. This would lead to change in the start time s'_1 and finish time $s'_{N'+1}$. Let the starting time of transmission s'_1 change to $s'_1 + x$ and the finish time $s'_{N'+1}$ change to $s'_{N'+1} + y$ for some $x, y > 0$ (note that y is dependent on x). We denote such a policy by vectors $\{\mathbf{p}'(x), \mathbf{s}'(x), N'(x)\}$.

Following Lemma 4, we can conclude that $(s'_{N'(x)+1}(x) - s'_1(x)) < (s'_{N'+1} - s'_1)$. We continue increasing x till either $u = p'_2(x)$ (in which case we change $l = s'_3(x)$) or $v = p'_{N'-1}(x)$ (where we change $r = s'_{N'-1}(x)$) or $s'_{N'(x)+1}(x)$ hits an epoch, say τ_j (we change $r = \tau_j$, $v \rightarrow \infty$ in this case). After this, we again start increasing x with changed definitions of l, r, u, v . We continue this process till $x = s_1 - s'_1$ or u becomes equal to v . Note that the value of x for which u becomes equal to v , would be greater than $(s_1 - s'_1)$, since policy $\{\mathbf{p}'(x), \mathbf{s}'(x), N'(x)\}$ shares at least one epoch with policy X , by arguments of previous paragraph. By maintaining these rules on l, r, u, v , we ensure that policy $\{\mathbf{p}'(x), \mathbf{s}'(x), N'(x)\}$ abides by structure (24)-(26), (28) and is feasible with energy constraint. Since $(s'_{N'(x)+1}(x) - s'_1(x))$ is decreasing with x , and $(s'_{N'(0)+1}(0) - s'_1(0)) = s'_{N+1} - s_1 \leq \Gamma_0$, the policy $\{\mathbf{p}'(x), \mathbf{s}'(x), N'(x)\}$ is also feasible with receiver time constraint. At $x = s_1 - s'_1$, we reach a policy such that $s'_1(x) = s_1$. For $x = s_1 - s'_1$, if $s'_{N'(x)+1}(x) \geq s_{N+1}$ then $s'_{N'+1} - s'_1 >$

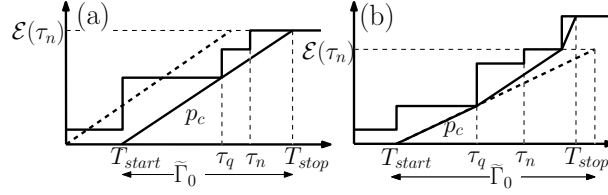


Fig. 5. Figure showing point τ_q .

$s'_{N'(x)+1}(x) - s'_1(x) \geq s_{N+1} - s_1 = \Gamma_0$ and policy Y is infeasible with receiver time constraint. If $s'_{N'(x)+1}(x) < s_{N+1}$, then we can follow the arguments presented in *Case2* to show that policy $\{p'(x), s'(x), N'(x)\}$ (at $x = s_1 - s'_1$) is infeasible, which in turn shows the infeasibility of policy Y . \square

IV. OPTIMAL OFFLINE ALGORITHM

In this section, we propose an offline algorithm OFF for Problem (14), and show that it satisfies the sufficiency conditions of Theorem 1. Algorithm OFF first finds an initial feasible solution via INIT_POLICY, and then iteratively improves upon it via PULL_BACK. Finally, QUIT produces the output.

Algorithm 1 OFF

- 1 **Input:** $\mathcal{E}(t), B_0, \Gamma_0$.
 - 2 $\{p, s, N\} \leftarrow \text{INIT_POLICY}(\mathcal{E}(t), B_0, \Gamma_0)$.
 - 3 $X \leftarrow \text{PULL_BACK}(\{p, s, N\})$.
 - 4 $\{p, s, N\} \leftarrow \text{QUIT}(X)$.
 - 5 **return** $\{p, s, N\}$.
-

A. INIT_POLICY

Idea: Initially, we find a constant power policy that is feasible and starts as early as possible. Also, we try to make it satisfy most of the sufficiency conditions of Theorem 1.

Step1: Identify the first energy arrival instant τ_n , so that using $\mathcal{E}(\tau_n)$ energy and Γ_0 time, B_0 or more bits can be transmitted with a constant power (say p_c), i.e. $\Gamma_0 g\left(\frac{\mathcal{E}(\tau_n)}{\Gamma_0}\right) \geq B_0$. Then solve for $\tilde{\Gamma}_0$,

$$\tilde{\Gamma}_0 g\left(\frac{\mathcal{E}(\tau_n)}{\tilde{\Gamma}_0}\right) = B_0, \quad p_c = \frac{\mathcal{E}(\tau_n)}{\tilde{\Gamma}_0}. \quad (32)$$

Step2: Find the earliest time T_{start} , such that transmission with power p_c from T_{start} for $\tilde{\Gamma}_0$ time is feasible with energy constraint (16). Set $T_{stop} = T_{start} + \tilde{\Gamma}_0$. Let τ_q be the *first epoch*, where $U(\tau_q) = \mathcal{E}(\tau_q^-)$ (Fig. 5). Lemma 6 shows that point τ_q thus found leads to a ‘good’ initial solution as, in every optimal solution total harvested energy till τ_q is used up at τ_q . This in-turn implies that $\tau_q \in s$, if $\{p, s, N\}$ is the optimal policy.

If $U(T_{stop}) = \mathcal{E}(T_{stop}^-)$ as shown in Fig. 5(a), then terminate INIT_POLICY with constant power policy p_c .

Otherwise, if $U(T_{stop}) < \mathcal{E}(T_{stop}^-)$, then modify the transmission after τ_q as follows. Set $\tilde{B}_0 = (T_{stop} - \tau_q)g(p_c)$, which denotes the number of bits left to be sent after time τ_q . Then apply Algorithm 1 of [5] from time τ_q to transmit \tilde{B}_0 bits in as minimum time as possible without considering the receiver on time constraint. Update T_{stop} , to where this policy ends. So, $U(T_{stop}) = \mathcal{E}(T_{stop}^-)$ from [5]. Since Algorithm 1 [5] is optimal, it takes minimum time ($= T_{stop} - \tau_q$) to transmit \tilde{B}_0 starting at time τ_q . However, using power p_c to transmit \tilde{B}_0 takes $(T_{start} + \tilde{\Gamma}_0 - \tau_q)$ time. Hence, $T_{stop} \leq (T_{start} + \tilde{\Gamma}_0)$. As $\tilde{\Gamma}_0 \leq \Gamma_0$ from (32), $(T_{stop} - T_{start}) \leq \Gamma_0$. This shows that solution thus found using Algorithm 1 [5], is indeed feasible with receiver time constraint (17). Now, output of INIT_POLICY is a policy that transmits at power p_c from T_{start} to τ_q , and after τ_q uses Algorithm 1 of [5].

Lemma 6. In every optimal solution, at energy arrival epoch τ_q defined in INIT_POLICY, $U(\tau_q) = \mathcal{E}(\tau_q^-)$.

Proof. We shall prove this by contradiction. For simplicity of notation let $R = T_{start}$ and $S = T_{stop}$ with T_{start}, T_{stop} being the start and finish time of constant power policy p_c defined in INIT_POLICY. First, we make the following claims:

Claim 1: Every optimal transmission policy begins transmission at or before time R .

Since, $S - R = \tilde{\Gamma}_0 \leq \Gamma_0$, by Lemma 5, if a transmission policy has to finish before S , it has to start before time $\max(S - \Gamma_0, 0) \leq \max(R, 0) = R$.

Claim 2: Every optimal transmission policy ends transmission at or before time S .

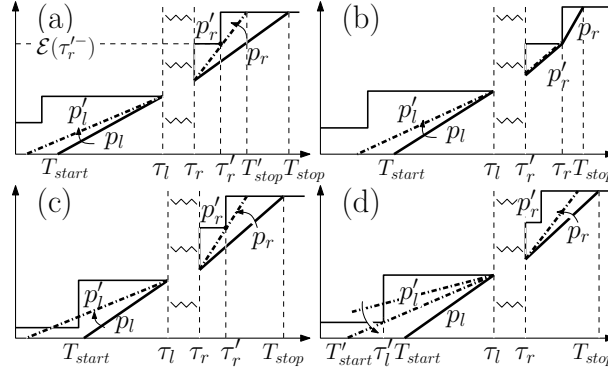


Fig. 6. Figures showing possible configurations in any iteration of the PULL_BACK. The solid line represents the transmission policy in the previous iteration and dash dotted lines are for the current iteration.

If it does not, then constant power policy p_c finishing at S will contradict its optimality.

Suppose we have an optimal transmission policy, say $X, \{p, s, N\}$, that does not exhaust all its energy at time τ_q i.e. $U(\tau_q) < \mathcal{E}(\tau_q^-)$. Then, by Lemma 3, it does not change its transmission power at τ_q . Let the transmission power of X be p_{j-1} at τ_q and p_{j-1} starts from s_{j-1} and goes till s_j . Now, $s_j < S$ by Claim 2. Further, power p_c exhausts all energy by τ_q . So,

$$p_c(\tau_q - R) = \mathcal{E}(\tau_q^-). \quad (33)$$

But, by constraint (16),

$$p_c(\tau_q - R) + p_c(s_j - \tau_q) \leq \mathcal{E}(s_j^-), \quad (34)$$

$$p_c(s_j - \tau_q) \stackrel{(a)}{\leq} \mathcal{E}(s_j^-) - \mathcal{E}(\tau_q^-), \quad (35)$$

$$p_c(s_j - \tau_q) < \mathcal{E}(s_j^-) - U(\tau_q) = p_{j-1}(s_j - \tau_q), \quad (36)$$

$$p_c < p_{j-1}, \quad (37)$$

where (a) follows from (33). If $j - 1 = 1$, then power at τ_q is the first transmission power p_1 . But then by (37), $p_1 > p_c$. By the definition of p_c , we must have $s_1 > R$, and this will contradict Claim 1.

So $j - 1 \geq 2$, which means that the power of transmission must change at least once between R and τ_q . By Lemma 3, X has used all energy by s_{j-1} and s_j . So, $p_j(\mathcal{E}(s_j^-) - \mathcal{E}(s_{j-1}^-))$ is the maximum energy available between time s_{j-1} and s_j . If $R < s_{j-1}$, then p_c (by (37)) uses more energy, than available between s_{j-1} and s_j , which is not possible. If $s_{j-1} \leq R$ then p_{j-1} uses more than maximum energy available (given by $p_c(\tau_q - R) = \mathcal{E}(\tau_q^-)$) between time R and τ_q , violating energy constraint (16).

Therefore, every optimal transmission policy must use all energy till epoch τ_q . □

Now that we have an initial feasible solution, we improve upon this policy iteratively as presented in PULL_BACK. But, before getting into the formal algorithm, we explain the procedure PULL_BACK through an example.

Example PULL_BACK: Assume that the starting feasible solution is given by the constant power policy, as shown by dotted line in Fig. 7 (a), where $\tau_q = \tau_2$. We first assign the following initial values for the initial feasible policy - transmission power left of τ_2 as $p_l = p_c$, power right of τ_2 as $p_r = p_c$, start time T_{start} stop time T_{stop} as start and stop time of constant policy power p_c , epoch at which p_l ends as $\tau_l = \tau_2$, epoch at which p_r starts as $\tau_r = \tau_2$. Now, we increase p_r , till it reaches p'_r which hits the boundary of energy feasibility at epoch τ_3 , as shown by the solid line in Fig 7 (a). Since, in total we need to transmit B_0 bits, the decrease in bits transferred by p_r to p'_r (RHS of (39)) is compensated by calculating appropriate p'_l according to the following equation, where LHS represents the increase in bits transmitted from p_l to p'_l .

$$\begin{aligned} & g(p'_l) \frac{\mathcal{E}(\tau_l^-)}{p'_l} - g(p_l)(\tau_l - T_{start}) \\ &= -g(p_r)(T_{stop} - \tau_r) + g(p'_r) \frac{\mathcal{E}(T'_{stop}) - \mathcal{E}(\tau_3^-)}{T'_{stop} - \tau_3}. \end{aligned} \quad (38)$$

Having got a feasible p'_l , as shown in Fig. 7 (a), we assign T'_{start} with the time at which transmission with power p'_l starts, T'_{stop} with time at which transmission with power p'_r finishes. τ'_r gets the value τ_3 and τ'_l remains same as $\tau_l = \tau_2$. Note that parameters $\{T'_{start}, T'_{stop}, \tau'_l, \tau'_r, p'_l, p'_r\}$ define the policy at the end of first iteration.

In the next iteration, the portion of transmission between $\tau'_l = \tau_2$ to $\tau'_r = \tau_3$ is not updated. In this iteration, we try to increase p'_r till it hits the feasibility equation (16) of energy. p'_r could virtually be increased to infinity. But transmission with

Algorithm 2 INIT_POLICY

```

1 Input:  $\mathcal{E}(t)$ ,  $B_0$ ,  $\Gamma_0$ 
2  $n = \arg \min_k \left( \left\{ \tau_k | \Gamma_0 g \left( \frac{\mathcal{E}(\tau_k)}{\Gamma_0} \right) \geq B_0 \right\} \right)$ .
3 Solve for  $\tilde{T} : \tilde{T} g \left( \frac{\mathcal{E}(\tau_n)}{\tilde{T}} \right) = B_0$ .
4  $p_c = \frac{\mathcal{E}(\tau_n)}{\tilde{T}}$ .
5  $q = \arg \min_{k \in [n]} (\{ \tau_k | IS\_FEASIBLE(\{p_c, p_c\}, \{\tau_k - \mathcal{E}(\tau_k^-)/p_c, \tau_k, \tau_k + (\mathcal{E}(\tau_n) - \mathcal{E}(\tau_k^-))/p_c\}, 2) == 1 \})$ .
6  $T_{start} = \tau_q - \frac{\mathcal{E}(\tau_q^-)}{p_c}$ ,  $T_{stop} = \tau_q + \frac{\mathcal{E}(\tau_n) - \mathcal{E}(\tau_q^-)}{p_c}$ .
7 if  $U(T_{stop}) < \mathcal{E}(\tau_n)$  then
8    $\tilde{B} = g(p_c)(T_{stop} - \tau_q)$ .
9    $\{p, s, N\} \leftarrow$  Apply Algorithm 1 in [5] to minimize transmission
      time of  $\tilde{B}$  bits after time  $\tau_q$  assuming a total of  $\mathcal{E}_q$ 
      amount of energy available at  $\tau_q$ .
10  return  $\{p_c, p\}, \{T_{start}, s\}, N + 1$ .
11 else
12  return  $\{p_c, p_c\}, \{T_{start}, \tau_q, T_{stop}\}, 2$ .
13 end if

```

$IS_FEASIBLE(p, s, N)$ returns 1 if policy $\{p, s, N\}$ is feasible and 0 otherwise.

infinite power for 0 time does not transmit any bits. So we assign $\tau_r'' = \tau_2$ and $p_r'' = \frac{\mathcal{E}(\tau_3^-) - \mathcal{E}(\tau_2^-)}{\tau_3 - \tau_2}$. With this change of p_r' to p_r'' , we again calculate p_l' which compensates the decrease in bits transferred after τ_r' . But the calculated p_l' becomes infeasible at τ_1 as shown in Fig. 7 (b). Hence, we set p_l' to the minimum feasible power $\frac{\mathcal{E}(\tau_2^-) - \mathcal{E}(\tau_1^-)}{\tau_2 - \tau_1}$ as shown in Fig. 7 (c). With this p_l' , we re-calculate p_r'' , so as to transmit B_0 bits in total. τ_l'' is assigned to τ_1 , τ_r'' remains τ_3 . T_{start}'' and T_{stop}'' are updated to values marked in Fig. 7 (c). The final policy at the end of second iteration is shown by solid line in Fig. 7 (c). Similarly, we continue to the third iteration, by improving the policy at the end of second iteration to finish earlier.

B. PULL_BACK

Now, we describe the iterative subroutine PULL_BACK whose input is policy $\{p, s, N\}$ output by INIT_POLICY.

Idea: Clearly, $\{p, s, N\}$, the output of INIT_POLICY, satisfies all but structure (27) of Theorem 1, since we cannot guarantee whether $(s_{N+1} - s_1) = \Gamma_0$ when $s_1 > 0$. So, the main idea of PULL_BACK is to increase the transmission duration from $(s_{N+1} - s_1) \leq \tilde{\Gamma}_0$, in INIT_POLICY, to Γ_0 in order to satisfy (27), while decreasing the finish time for reaching the optimal solution. To achieve this, we utilize the structure presented in Lemma 4 and iteratively increase the last transmission power p_N , and decrease the first transmission power p_1 .

Initialize $\tau_l = s_2$, $\tau_r = s_N$, $p_l = p_1$, $p_r = p_N$, $T_{start} = s_1$, $T_{stop} = s_{N+1}$. In any iteration, τ_l and τ_r are assigned to the first and last energy arrival epochs, where $U(\tau_l) = \mathcal{E}(\tau_l^-)$ and $U(\tau_r) = \mathcal{E}(\tau_r^-)$. p_l and p_r are the constant transmission powers before τ_l and after τ_r , respectively. We reuse the notation τ here, because τ_l and τ_r will occur at energy arrival epochs from Lemma 3. T_{start} and T_{stop} are the start and finish time of the policy, found in any iteration. $\tau_l, \tau_r, p_l, p_r, T_{start}, T_{stop}$ get updated to $\tau_l', \tau_r', p_l', p_r', T_{start}', T_{stop}'$ over an iteration. In any iteration, only one of τ_l or τ_r gets updated, i.e., either $\tau_l' = \tau_l$ or $\tau_r' = \tau_r$. Further, PULL_BACK ensures that *transmission powers between τ_l and τ_r do not get changed* over an iteration. Fig. 6 shows the possible updates in an iteration of PULL_BACK.

Step1, Updation of τ_r, p_r : Initialize $p_r' = p_r$ and increase p_r' till it hits the boundary of energy constraint (16), say at $(\tau_r', \mathcal{E}(\tau_r'^-))$ as shown in Fig. 6(a). The last epoch where p_r' hits (16) is set to τ_r' . So, $U(\tau_r') = \mathcal{E}(\tau_r'^-)$. Set T_{stop}' to where power p_r' ends. Calculate p_l' such that decrease in bits transmitted due to change from p_r to p_r' is compensated by increasing p_l to p_l' , via

$$\begin{aligned}
& g(p_r)(T_{stop} - \tau_r) - g(p_r')(T_{stop}' - \tau_r') \\
& = g(p_l') \frac{\mathcal{E}(\tau_l'^-)}{p_l'} - g(p_l)(\tau_l - T_{start}).
\end{aligned} \tag{39}$$

Suppose, p_r can be increased till infinity without violating (16), as shown in Fig. 6(b). This happens when there is no energy arrival between τ_r and T_{stop} . In this case, set p_r' to the transmission power at τ_r^- . Set τ_r' as the epoch where p_r' starts, and T_{stop}' to τ_r . Calculate p_l' similar to (39).

Step2, Updation of τ_l, p_l : If p_l' obtained from Step1 is feasible, as shown in Fig. 6(a), set $T_{start}' = \tau_l - \frac{\mathcal{E}(\tau_l'^-)}{p_l'}$, $\tau_l' = \tau_l$. Proceed to Step3. Otherwise, if p_l' is not feasible, as shown in Fig. 6(c), the changes made to τ_r', p_r' in Step1 are discarded. As shown in Fig. 6 (d), p_l' is increased from its value in Step1 until it becomes feasible. τ_l' is set to the first epoch where $U(\tau_l') = \mathcal{E}(\tau_l'^-)$.

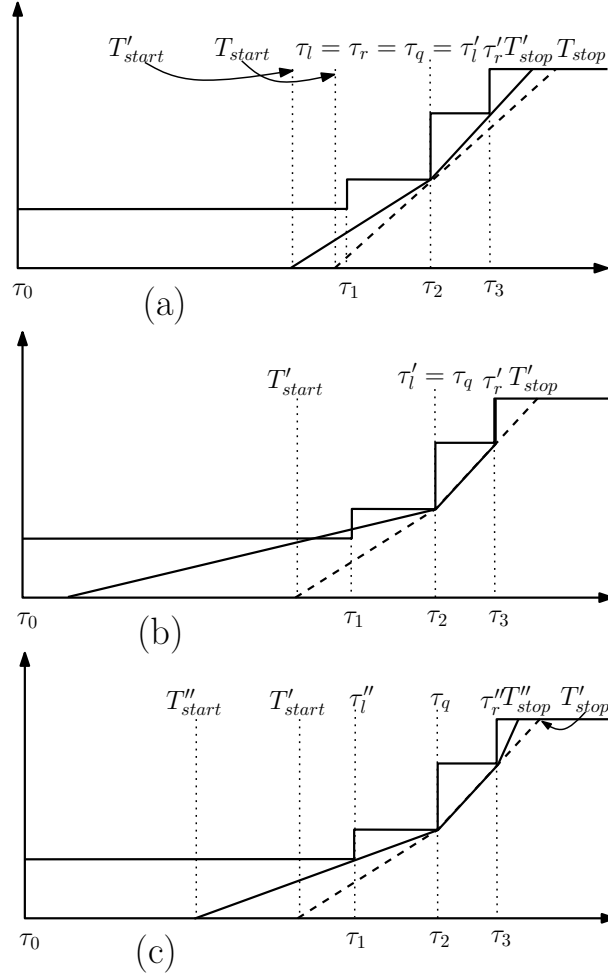


Fig. 7. Figures showing (a) first and (c) second iteration of the PULL_BACK through an example. (b) represents an intermediate step in second iteration. In this diagram, the dashed line represent previous iteration policy and solid line is the present iteration policy.

Similar to *Step1*, calculate p'_r such that the increase in bits transmitted due to change of p_l to p'_l is compensated, and update T'_{stop} accordingly. Set $\tau'_r = \tau_r$. Proceed to *Step3*.

Step3, Termination condition: If $T'_{stop} - T'_{start} \geq \Gamma_0$ or $T'_{start} = 0$, then terminate PULL_BACK. Otherwise, update $\tau_l, \tau_r, p_l, p_r, T_{start}, T_{stop}$ to $\tau'_l, \tau'_r, p'_l, p'_r, T'_{start}, T'_{stop}$ receptively and GOTO *Step1*.

By design of PULL_BACK, we know that the finish time decreases at every iteration. Next, in Lemma 7, we show that transmission time increases with each iteration of PULL_BACK. This is used in Lemma 8 to establish a bound on the running time of PULL_BACK.

Lemma 7. *Transmission time $(T_{stop} - T_{start})$ monotonically increases over each iteration of PULL_BACK.*

Proof. In any iteration of PULL_BACK, the possible valid configurations can be one of the three shown in Fig. 6 (a), (b), (d). Since it is too verbose to describe the three possible cases, we refer to Fig. 6. Note that $\mathcal{E}(T_{stop}^-) = \mathcal{E}(T_{stop}^{'-})$ in (a), (d). In case (b), we can assume that $T'_{stop} = \tau_r^+$ and transmission continues beyond τ_r , but with infinite power. Since transmitting with infinite power for 0 time does not transmit any bits, we would transmit the same number of bits, as we did prior to this modification. So, $\mathcal{E}(T_{stop}^-) = \mathcal{E}(T_{stop}^{'-})$ in (d) as well. Thus, the transmission policy for two consecutive iterations satisfy the conditions of Lemma 4 (with $\beta \rightarrow \infty$ for case (d)) and therefore, $(T_{stop} - T_{start})$ increases across constitutive iterations of PULL_BACK. \square

Lemma 8. *Worst case running time of PULL_BACK is linear with respect to the number of energy harvests before finish time of INIT_POLICY.*

Proof. Since, in an iteration of PULL_BACK, either τ_r or τ_l updates, the number of iterations is bounded by the values attained by τ_l , in addition to that of τ_r . Initially, $\tau_l \leq \tau_q$ and $\tau_r \geq \tau_q$. As τ_l is non-increasing across iterations, $\tau_l \leq \tau_q$ throughout. Assume that τ_r remains greater than or equal to τ_q across INIT_POLICY. Then, both τ_l and τ_r can at max attain all τ_i 's less than finish time of initial feasible policy. Hence, we are done.

It remains to show that $\tau_r \geq \tau_q$. τ_n is defined as the first energy arrival epoch with which B_0 or more bits can be transmitted in Γ_0 time and $\tau_q \leq \tau_n$, by definition. So, when T_{stop} becomes $\leq \tau_n$ or τ_q , then transmission time, $(T_{stop} - T_{start})$, should be $> \Gamma_0$. But, in the initial iteration $(T_{stop} - T_{start}) \leq \Gamma_0$ and $(T_{stop} - T_{start})$ increases monotonically, from Lemma 7. Hence, PULL_BACK will terminate before T_{stop} (and therefore τ_r) decreases beyond τ_q . \square

Algorithm 3 PULL_BACK

```

1 Input:  $\{p, s, N\} \leftarrow \text{INIT\_POLICY}$ 
2 Initialization:  $\tau_l = s_2, \tau_r = s_N, T_{start} = s_1, T_{stop} = s_{N+1}, p_l = p_1, p_r = p_N, \text{control} = 0.$ 
3 Delete  $s.first$ , Delete  $s.last$ , Delete  $p.first$ , Delete  $p.last$ .
4 while  $(T_{stop} - T_{start} < \Gamma_0)$  and  $(T_{start} > 0)$  do
5    $\{\tau'_l, \tau_r, T'_{start}, T'_{stop}, p'_l, p'_r, p', s'\}$ 
6   if  $\{i : \tau_r < \tau_i < T_{stop}\} = \emptyset$  then
7      $B_l = g(p_r)(T_{stop} - \tau_r) + g(p_l)(\tau_l - T_{start}), \text{control} = 1.$ 
8   else
9      $j = \arg \min_{i: \tau_r < \tau_i < T_{stop}} \mathcal{P}(\tau_r, \tau_i).$ 
10     $B_l = g(p_r)(T_{stop} - \tau_r) + g(p_l)(\tau_l - T_{start}) - g(\mathcal{P}(\tau_r, \tau_j)) \left( \frac{\mathcal{E}(T_{stop}^-) - \mathcal{E}(\tau_r^-)}{\mathcal{P}(\tau_r, \tau_j)} \right).$ 
11  end if
12  Solve for  $\tilde{p}$  in  $0 < \tilde{p} < p_l$ :  $\frac{\mathcal{E}(\tau_l^-)}{\tilde{p}} g(\tilde{p}) = B_l.$ 
13  if  $\tilde{p}$  exists then
14     $istrue = IS\_FEASIBLE(\tilde{p}, \{\tau_l - \mathcal{E}(\tau_l^-)/\tilde{p}, \tau_l\}, 1).$ 
15  else  $istrue = 0.$ 
16  end if
17  if  $istrue == 1$  then
18     $p_l = \tilde{p}, T_{start} = \tau_l - \mathcal{E}(\tau_l^-)/p_l.$ 
19    if  $\text{control} = 0$  then
20       $p_r = \mathcal{P}(\tau_r, \tau_j), T_{stop} = \tau_r + \frac{\mathcal{E}(T_{stop}^-) - \mathcal{E}(\tau_r^-)}{\mathcal{P}(\tau_r, \tau_j)}.$ 
21       $\tau_r = \tau_j, p.append(\mathcal{P}(\tau_r, \tau_j)), s.append(\tau_j).$ 
22    else
23       $T_{stop} = \tau_r.$ 
24      if  $p \neq \emptyset$  then
25        Delete  $s.last, \tau_r = s.last, p_r = p.last$ , Delete  $p.last$ .
26      end if
27       $\text{control} = 0.$ 
28    end if
29  else
30     $k = \arg \max_{i: \max((\tau_l - \mathcal{E}(\tau_l^-)/\tilde{p}), \tau_0) \leq \tau_i < \tau_l} \mathcal{P}(\tau_i, \tau_l).$ 
31     $B_r = g(p_r)(T_{stop} - \tau_r) + g(p_l)(\tau_l - T_{start}) - g(\mathcal{P}(\tau_k, \tau_l)) \cdot \left( \frac{\mathcal{E}(\tau_l^-)}{\mathcal{P}(\tau_k, \tau_l)} \right)$ 
32     $p_l = \mathcal{P}(\tau_k, \tau_l), T_{start} = \tau_l - \mathcal{E}(\tau_l^-)/\mathcal{P}(\tau_k, \tau_l).$ 
33     $\tau_l = \tau_k, p.prepend = p_l, s.prepend = \tau_k.$ 
34    Solve for  $p_r$ :  $\frac{\mathcal{E}(T_{stop}^-) - \mathcal{E}(\tau_r^-)}{p_r} g(p_r) = B_r.$ 
35     $T_{stop} = \tau_r + (\mathcal{E}(T_{stop}^-) - \mathcal{E}(\tau_r^-))/p_r.$ 
36  end if
37 end while
38 return  $\{\tau'_l, \tau'_r, T'_{start}, T'_{stop}, p'_l, p'_r, p', s', T_{start}, T_{stop}\}.$ 

```

$$\mathcal{P}(\tau_i, \tau_j) = \frac{\mathcal{E}(\tau_j^-) - \mathcal{E}(\tau_i^-)}{\tau_j - \tau_i}.$$

The third and final subroutine of OFF is QUIT.

C. QUIT

If $T'_{start} = 0$ and $T'_{stop} - T'_{start} \leq \Gamma_0$ upon PULL_BACK's termination, then PULL_BACK's policy at termination is output. Note that structure (27) holds for this policy. Otherwise, if $T'_{stop} - T'_{start} > \Gamma_0$ (which happens for the first time across iterations of PULL_BACK), then we know that in penultimate step $T_{stop} - T_{start} < \Gamma_0$. Hence, we are looking for a policy that starts in $[T_{start}, T'_{start}]$ and ends in $[T_{stop}, T'_{stop}]$, whose transmission time is equal to Γ_0 . We solve for x, y (let the solution be x^*, y^*),

$$\begin{aligned}
(\tau_l - x) g\left(\frac{\mathcal{E}(\tau_l^-)}{\tau_l - x}\right) + (y - \tau_r) g\left(\frac{\mathcal{E}(T_{stop}^-)}{y - \tau_r}\right) \\
= g(p_l)(\tau_l - T_{start}) + g(p_r)(T_{stop} - \tau_r),
\end{aligned} \tag{40}$$

$$y - x = \Gamma_0. \tag{41}$$

Algorithm 4 QUIT

```

1 Input:
2  $\{\tau'_l, \tau'_r, T'_{start}, T'_{stop}, p'_l, p'_r, \mathbf{p}', \mathbf{s}', T_{start}, T_{stop}\} \leftarrow \text{PULL\_BACK}$ .
3 if  $(T_{start} - T_{stop}) > \Gamma_0$  then
4    $T = \Gamma_0 - (\tau'_r - \tau'_l)$ ,  $B = B_0 - \sum_i g(p'_i)(s'_{i+1} - s'_i)$ .
5   Solve for  $x$ :
6      $p'_l = \frac{\mathcal{E}(\tau'^{-}_l)}{x}$ ,  $T'_{start} = \tau'_l - x$ .
7      $p'_r = \frac{\mathcal{E}(T'^{-}_{stop}) - \mathcal{E}(\tau'^{-}_r)}{T - x}$ ,  $T'_{stop} = \tau'_r + T - x$ .
8      $\mathbf{p}'.\text{prepend}(p'_l)$ ,  $\mathbf{s}'.\text{prepend}(T'_{start})$ ,  $\mathbf{p}'.\text{append}(p'_r)$ ,  $\mathbf{s}'.\text{append}(T'_{stop})$ .
9     return  $\{\mathbf{p}', \mathbf{s}', \text{number of elements in } \mathbf{p}'\}$ .
10 else
11    $\mathbf{p}.\text{prepend}(p_l)$ ,  $\mathbf{s}.\text{prepend}(T_{start})$ ,  $\mathbf{p}.\text{append}(p_r)$ ,  $\mathbf{s}.\text{append}(T_{stop})$ .
12   return  $\{\mathbf{p}, \mathbf{s}, \text{number of elements in } \mathbf{p}\}$ .
13 end if

```

At penultimate iteration, $(x, y) = (T_{start}, T_{stop})$, (40) is satisfied and $y - x < \Gamma_0$. At $(x, y) = (T'_{start}, T'_{stop})$, as $\mathcal{E}(T'^{-}_{stop}) = \mathcal{E}(T'_{stop})$, (40) is satisfied and $y - x > \Gamma_0$. So, there must exist a solution (x^*, y^*) to (40), where $x^* \in [T'_{start}, T_{start}]$, $y^* \in [T'_{stop}, T_{stop}]$ and $y^* - x^* = \Gamma_0$, for which, (27) holds. Output with this policy which starts at x^* and ends at y^* .

Now, we state Theorem 2 which proves the optimality of Algorithm OFF.

Theorem 2. *The transmission policy proposed by Algorithm OFF is an optimal solution to Problem (14).*

Proof. We show that Algorithm OFF satisfies the sufficiency conditions of Theorem 1. To begin with, we prove that the power allocations satisfy (25), by induction. First we establish the base case that INIT_POLICY's output satisfies (25). If INIT_POLICY returns the constant power policy p_c from time T_{start} to T_{stop} , then clearly the claim holds.

Otherwise, INIT_POLICY applies Algorithm 1 from [5] with $\tilde{B} = B_0 - g(p_c)(\tau_q - T_{start})$ bits to transmit after time τ_q . Algorithm 1 from [5] ensures that transmission powers are non-decreasing after τ_q . So we only need to prove that the transmission power p_c between time T_{start} and τ_q is less than or equal to the transmission power just after τ_q (say p_q), via contradiction. Assume that $p_q < p_c$. Let transmission with p_q end at an epoch $\tau_{q'}$, where $U(\tau_{q'}) = \mathcal{E}(\tau_{q'}^-)$ from [5]. The energy consumed between time τ_q to $\tau_{q'}$ with power p_c is,

$$p_c(\tau_{q'} - \tau_q) > p_q(\tau_{q'} - \tau_q) \stackrel{(a)}{=} \mathcal{E}(\tau_{q'}^-) - \mathcal{E}(\tau_q^-), \quad (42)$$

where (a) follows from $U(\tau_q) = \mathcal{E}(\tau_q^-)$. Further, the maximum amount of energy available for transmission between τ_q and $\tau_{q'}$ is $(\mathcal{E}(\tau_{q'}^-) - \mathcal{E}(\tau_q^-))$. By (42), transmission with p_c uses more than this energy and therefore it is infeasible between time τ_q and $\tau_{q'}$. But, by definition of \tilde{p}_c , transmission with power p_c is feasible till time $(T_{start} + \tilde{\Gamma}_0)$. Also, $\tau_{q'} \leq T_{stop}$ by definition of $\tau_{q'}$ and $T_{stop} \leq (T_{start} + \tilde{\Gamma}_0)$. So, power p_c must be feasible till $\tau_{q'}$ and we reach a contradiction.

Now, we assume that the transmission powers output from PULL_BACK are non-decreasing till its n^{th} iteration. Therefore, as transmission powers between τ_l and τ_r does not change over an iteration, powers would remain non-decreasing in the $(n+1)^{th}$ iteration if we show that $p'_l < p_l$ and $p'_r > p_r$. In any iteration, by definition, either τ_l or τ_r updates. Assume τ_l gets updated to τ'_l , p_l to p'_l , p_r to p'_r and τ_r remains same, shown Fig. 6(d) (when τ_r updates, the proof follows similarly). Then we are certain that $p'_r > p_r$ by algorithmic steps. So from n^{th} to $(n+1)^{th}$ iteration, the number of bits transmitted after τ_r should decrease. Thus, the number of bits transmitted before τ_l must be increasing. This implies $p'_l \leq p_l$ and this completes the proof for transmission powers being non-decreasing at the end of every iteration of PULL_BACK.

Next, we show that QUIT outputs a policy with non-decreasing transmission powers. Let the policy being output by QUIT be X , $\{\mathbf{p}, \mathbf{s}, N\}$. Let the start and finish time of the policy at the penultimate iteration of PULL_BACK (say Y) be T_{start} and T_{stop} , respectively. From the algorithmic design of PULL_BACK, we know that Y is identical to X from time s_2 to s_N . Also, since Y is a policy from PULL_BACK, it has non-decreasing transmission powers. Thus, we can write the power profile of Y as $\{\frac{\mathcal{E}(s_2'^{-})}{s_2 - T_{start}}, p_2, p_3, \dots, p_{N-1}, \frac{\mathcal{E}(s_{N+1}'^{-})}{T_{stop} - s_N}\}$, where

$$\frac{\mathcal{E}(s_2'^{-})}{s_2 - T_{start}} \leq p_2 \leq \dots \leq p_{N-1} \leq \frac{\mathcal{E}(s_{N+1}'^{-})}{T_{stop} - s_N}. \quad (43)$$

Hence, in order to prove monotonicity of \mathbf{p} , we only need to show $p_1 \leq p_2$ and $p_{N-1} \leq p_N$. From QUIT, recall that $s_1 = x^* \leq T_{start}$ and $s_{N+1} = y^* \leq T_{stop}$. Thus, $p_1 = \frac{\mathcal{E}(s_2'^{-})}{s_2 - s_1} \leq \frac{\mathcal{E}(s_2'^{-})}{s_2 - T_{start}} \stackrel{(a)}{\leq} p_2$ and $p_N = \frac{\mathcal{E}(s_{N+1}'^{-})}{s_{N+1} - s_N} \geq \frac{\mathcal{E}(s_{N+1}'^{-})}{T_{stop} - s_N} \stackrel{(a)}{\geq} p_{N-1}$, where (a) follows from (43).

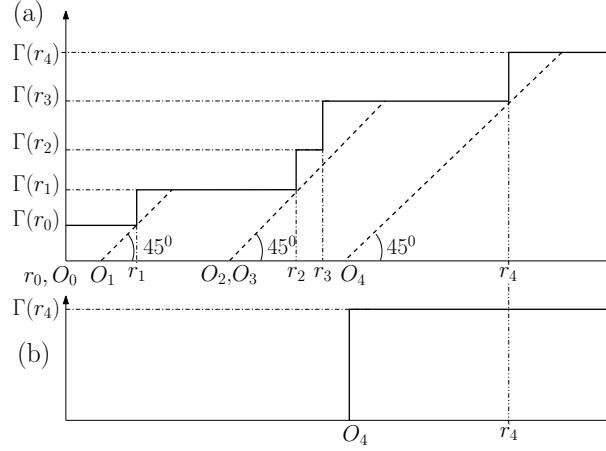


Fig. 8. (a) Figure showing O_i 's which represent the first time instances at which the receiver can be kept on continuously for $\Gamma(r_i)$ time. Note that O_2 and O_3 coincide in this example. (b) Energy harvesting profile at the receiver for problem $\text{OFF}(O_4)$.

Hence, transmission powers output by OFF are non-decreasing and satisfy (25). Since OFF transmits equal number of bits ($=B_0$) throughout INIT_POLICY, PULL_BACK and QUIT, it satisfies (24). Clearly, (26) is maintained throughout OFF, and by arguments presented at end of QUIT, we know that OFF satisfies (27).

Now consider structure (28). As τ_q is present in INIT_POLICY, the only way τ_q cannot be part of the policy (say $\{p, s, N\}$) in an iteration of PULL_BACK, i.e. $\tau_q \notin s$, is when τ_r decreases beyond τ_q . But $\tau_r \geq \tau_q$ as shown in Lemma 8. So, the policy output by OFF includes τ_q . To conclude, OFF satisfies (24)-(28), and hence is an optimal algorithm. \square

Discussion: In this section, we solved the special case of (9), when there is only energy arrival at the receiver. Even this special case is hard, compared to having receiver powered by conventional energy source. We proposed a three phase iterative algorithm, where in first we come up with a reasonable feasible solution and then improve upon it in the next two phases until it satisfies the sufficient conditions for the optimal solution. We use this solution of the special case as a building block to solve the general problem (9) in next section.

V. OFFLINE ALGORITHM FOR RECEIVER WITH MULTIPLE ENERGY ARRIVALS

We now consider solving the general problem (9) in the offline setting, when receiver harvests energy multiple times. Our approach to solve problem (9) is to use the algorithm OFF repeatedly. Corresponding to every receiver 'time' arrival of Γ_i at r_i , let O_i be the earliest time instant such that the receiver can be kept on continuously, without any break, from time O_i to $O_i + \Gamma(r_i)$ (see Fig. 8 (a)). It can be easily seen that the receiver will exhaust all its available energy (or attain the boundary of (12)) at atleast one receiver 'time' arrival epoch when it is kept on from O_i to $O_i + \Gamma(r_i)$. If not, then we can start the receiver slightly earlier than O_i and keep it on for $\Gamma(r_i)$ time without violating constraint (12), which is contradictory to our definition of O_i . For example, in Fig. 8 (a), when the receiver turns on from O_1 , it exhausts all its energy at r_1^- .

Let

$$i_0 = \min \left\{ i : \lim_{t \rightarrow \infty} \Gamma(r_i) g \left(\frac{\mathcal{E}(t)}{\Gamma(r_i)} \right) \geq B_0 \right\}, \quad (44)$$

i.e. i_0 defines the earliest energy arrival time r_i at the receiver such that the time ($\Gamma(r_i)$) for which the receiver can stay on starting from r_i is sufficient to transmit the B_0 bits by the transmitter eventually, even if no more energy arrives at the receiver.

Lemma 9. *If there is a solution to problem (9), then $i_0 < \infty$.*

Proof. Let the finish time of any feasible solution F for (9) be T . Then by time T , the maximum energy used by F to transmit B_0 bits at the transmitter is $\mathcal{E}(T)$ and the receiver is on for at most time $\Gamma(T)$. Let the last energy arrival at the transmitter and the receiver before time T be τ_{end} and r_{end} . Then, $i_0 \leq \max\{\tau_{end}, r_{end}\}$, since starting from time $\max\{\tau_{end}, r_{end}\}$, one can transmit B_0 bits for function $g(\cdot)$ using energy $\mathcal{E}(T)$ at the transmitter in receiver time of $\Gamma(T)$ without any break. \square

Now, for the sake of applying algorithm OFF in multiple receiver energy arrivals regime, we introduce a new optimization problem, called $\text{OFF}(O_i)$, for $i \geq i_0$.

$\text{OFF}(O_i)$ is defined under the following energy harvesting profile - the receiver has only one 'time' arrival of $\Gamma(r_i)$, the accumulated receiver time till r_i in problem (9), at time O_i (see Fig. 8 (b) for $i = 4$). The transmitter energy harvesting profile remains same as $\mathcal{E}(t)$, $\forall t \in [0, \infty)$. The formal description of problem $\text{OFF}(O_i)$ is as follows.

$$\min_{\{\mathbf{p}, \mathbf{s}, N\}, T=s_{N+1}} T \quad (45)$$

$$\text{subject to } B(T) = B_0, \quad (46)$$

$$U(t) \leq \mathcal{E}(t) \quad \forall t \in [O_i, T], \quad (47)$$

$$C(t) \leq \Gamma(r_i) \quad \forall t \in [O_i, T]. \quad (48)$$

$$C(t) = 0, U(t) = 0 \quad \forall t \in [0, O_i], \quad (49)$$

where $C(t)$ is defined in (13).

Since problem (45) has only one energy arrival at the receiver, we can use algorithm OFF to solve the problem of transmitting B_0 bits under this energy harvesting profile. With origin shifted to O_i , optimization problem (45) is similar to problem (14).

From Lemma 9, it is clear that if there is a solution to problem (9), then $\forall i \geq i_0$ there is a solution for $\text{OFF}(O_i)$. Let the optimal policy returned by solving $\text{OFF}(O_i)$ be denoted by X_i . Moreover, it's worthwhile remembering that X_i is also a feasible solution to (9). We have introduced $\text{OFF}(O_i)$ to break the complex problem (9) into simpler single receiver 'time' arrival problems $\text{OFF}(O_i)$ that can be solved using OFF. Lemma 10 states that the optimal solution to problem (9) is one of the X_i 's.

Also, following similar procedure as described in Lemma 2, we can show that there always exists an optimal solution to problem (9) with no breaks in transmission. So in the rest of the paper whenever we refer to the optimal solution of problem (9), we consider the one without breaks in transmission.

Lemma 10. *The optimal solution to problem (9) is policy X_i for some i .*

Proof. We shall prove this by contradiction. Assume that the optimal solution to problem (9) is given by policy Y , $\{\mathbf{p}, \mathbf{s}, N\}$, and none of the X_i 's are optimal to problem (9). Let $O_k \leq s_1 < O_{k+1}$ for some k . By definition of O_{k+1} , all policies starting before O_{k+1} must have transmission time less than $\Gamma(r_{k+1})$, and therefore the transmission time of Y ($s_{N+1} - s_1$) $\leq \Gamma(r_{k+1}) = \Gamma(r_k)$. Let X_k (solution of $\text{OFF}(O_k)$) be denoted by $\{\mathbf{p}', \mathbf{s}', N'\}$. Now, since the transmission time of Y is less than or equal to $\Gamma(r_k)$ and its start time is greater than O_k , policy Y is a feasible solution to $\text{OFF}(O_k)$. This implies $k \geq i_0$ and also,

$$s'_{N'+1} \leq s_{N+1}. \quad (50)$$

Both X_k and Y are feasible policies to problem (9), and Y is optimal to problem (9) from our assumption, whereas X_k is not. Therefore,

$$s_{N+1} < s'_{N'+1}. \quad (51)$$

Hence, (51) contradicts (50). \square

From Lemma 10, to solve (9) we need to identify the right index i for which X_i is optimal. Let the optimal policy for problem (9) be denoted by X_{i^*} . Next, Lemma 11 states that if X_{i^*} , the optimal policy to $\text{OFF}(O_{i^*})$, is also optimal to problem (9), then it must begin transmission before O_{i^*+1} .

Lemma 11. *The optimal policy to problem (9), X_{i^*} denoted by $\{\mathbf{p}, \mathbf{s}, N\}$, has $s_1 \leq O_{i^*+1}$.*

Proof. We prove it by contradiction. Let $s_1 > O_{i^*+1}$. Now, consider X_{i^*+1} ($\{\mathbf{p}', \mathbf{s}', N'\}$), the optimal policy to $\text{OFF}(O_{i^*+1})$ and X_{i^*} ($\{\mathbf{p}, \mathbf{s}, N\}$). Since $s_{N+1} - s_1 \leq \Gamma(r_{i^*})$ and $s_1 > O_{i^*+1}$, X_{i^*} is a feasible solution to $\text{OFF}(O_{i^*+1})$. Now, both X_{i^*} and X_{i^*+1} are feasible solutions to $\text{OFF}(O_{i^*+1})$ and X_{i^*+1} is optimal with respect to $\text{OFF}(O_{i^*+1})$. So, $s'_{N'+1} \leq s_{N+1}$.

But on the other hand, both X_{i^*} and X_{i^*+1} are feasible solutions to problem (9) and X_{i^*} is optimal with respect to problem (9). This implies that $s_{N+1} \leq s'_{N'+1}$. From the above arguments we can conclude that the only possibility is $s_{N+1} = s'_{N'+1}$.

So, both X_{i^*} and X_{i^*+1} are optimal with respect to $\text{OFF}(O_{i^*+1})$. By Theorem 1, optimal solution to $\text{OFF}(O_{i^*+1})$ is unique (optimal solution without breaks in transmission) and therefore, X_{i^*} and X_{i^*+1} have to be exactly identical. This would imply $s'_1 = s_1 > O_{i^*+1}$. Hence, by Lemma 5 on problem $\text{OFF}(O_{i^*+1})$, $s'_{N'+1} - s'_1 = \Gamma(r_{i^*+1})$. Also, $s_{N+1} - s_1 \leq \Gamma(r_{i^*})$ by the receiver 'time' constraint in problem $\text{OFF}(O_{i^*})$ and $\Gamma(r_{i^*}) < \Gamma(r_{i^*+1}) + \Gamma_{i^*+1} = \Gamma(r_{i^*+1})$. So, $s_{N+1} - s_1 < s'_{N'+1} - s'_1$ and this contradicts the identity of policies X_{i^*} and X_{i^*+1} . \square

Lemma 12 gives us a sufficient condition under which we can compute X_{i^*} . It establishes that the optimal policy to problem (9) is X_{i^*} , where i^* is the minimum i for which policy X_i 's start time is before O_{i+1} .

Lemma 12. *The optimal policy to problem (9) is X_{i^*} where*

$$i^* = \min \{i : s_1 \leq O_{i+1}, X_i \equiv \{\mathbf{p}, \mathbf{s}, N\}\}.$$

Proof. We will prove this by contradiction. Let j denote the minimum i for which policy X_i 's start time is less than or equal to O_{i+1} and let X_j be not optimal for problem (9). X_{i^*} being the optimal solution to problem (9), satisfies Lemma 11 and so $i^* > j$. Let X_j be denoted by $\{\mathbf{p}', \mathbf{s}', N'\}$ and X_{i^*} by $\{\mathbf{p}, \mathbf{s}, N\}$. Since X_{i^*} is the optimal policy to problem (9), we have

$$s_{N+1} \leq s'_{N'+1}. \quad (52)$$

Also, $s_1 \geq O_{i^*}$ by definition of X_{i^*} , and $O_{i^*} \geq O_{j+1}$ since $i^* > j$ and O_i 's are non decreasing with respect to i . Moreover, $s'_1 \leq O_{j+1}$ by definition of j . This would imply $s'_1 \leq s_1$. Hence, using (52), we can write

$$s_{N+1} - s_1 \leq s'_{N'+1} - s'_1. \quad (53)$$

From constraints of problem $\text{OFF}(O_j)$, $s'_{N'+1} - s'_1 \leq \Gamma(r_j)$. Combining this with (53), $s_{N+1} - s_1 \leq \Gamma(r_j)$. So, X_{i^*} is a feasible solution to $\text{OFF}(O_j)$. This would imply $s'_{N'+1} \leq s_{N+1}$ on account of optimality of X_j with respect to $\text{OFF}(O_j)$. When combined with (52), we have $s_{N+1} = s'_{N'+1}$. Therefore, X_j , having same finish time with X_{i^*} , is also a optimal policy to problem (9). But as we have shown earlier, the optimal policy is unique. Hence we get a contradiction on our assumption on X_j . \square

Now, we describe the algorithm OFFM to solve problem (9).

Algorithm OFFM:

Initialization: Let $i = i_0$, where i_0 is defined in (44).

Step1: Find policy X_i as solution to $\text{OFF}(O_i)$ using algorithm OFF.

Step2: If start time of X_i is less than or equal to O_{i+1} then output policy X_i as the optimal policy and terminate. If not, then increment i to $i + 1$ and go to *Step1*.

Theorem 3 stated below establishes the optimality of algorithm OFFM.

Theorem 3. *Algorithm OFFM returns optimal solution to problem (9).*

Proof. By Lemma 10, we know that the solution to problem 2 has to be a policy X_i for some i . Further, from Lemmas 11 and 12 we can conclude that the smallest index i for which X_i satisfies the condition of having its start time before O_{i+1} is the optimal solution.

As X_{i^*} is the optimal solution to problem (9), $i^* \geq i_0$, where i_0 is defined in (44). Since OFFM iteratively finds policy X_i for every value of $i \geq i_0$, it will definitely terminate with X_{i^*} in less than i^* number of iterations. Thus, Algorithm OFFM returns an optimal solution to problem (9). \square

Discussion: In this section, we derived the structure of the optimal power transmission profile in the offline setting, and derived an algorithm that satisfies the optimal structure. The main idea presented in this section is that the problem with multiple energy harvests at the receiver can be broken down into simpler problems, where there is only one energy harvest at the receiver. This hierarchical structure simplifies the complexity of the algorithm as well as provides us with an elegant method to construct a solution. As far as we know, such a hierarchical structure has not been discovered for other related energy harvesting problems.

VI. ONLINE ALGORITHM

In this section, we consider solving Problem (9) in the more realistic online scenario, where the transmitter and the receiver are assumed to have only causal information about energy arrivals, and both have infinite battery capacities. To consider the most general model, even the distribution of future energy arrivals is unknown at both the transmitter and the receiver.

Let $B_{\text{rem}}(t)$ and $E_{\text{rem}}(t)$ denote the remaining number of bits to be transmitted, and energy left at the transmitter, at any time t , respectively, for the online algorithm. In place of $\{p, s, N\}$ for the offline case, we use the notation $\{l, b, M\}$ to denote an online algorithm, with identical definitions. Thus, l_i power is transmitted between time b_i and b_{i+1} , and end time is b_{M+1} . Let σ be the set of all possible energy arrival sequences at the transmitter, ρ be the set of all time arrival sequences at the receiver and \mathbf{A} be the set of all online algorithms to solve (9). Then the competitive ratio is given by

$$r = \min_{A \in \mathbf{A}} \max_{\sigma \in \sigma, \rho \in \rho} \frac{T_A(\sigma, \rho)}{T_O(\sigma, \rho)}, \quad (54)$$

where $T_A(\sigma, \rho)$ and $T_O(\sigma, \rho)$ are the finish times taken by the online algorithm A and the optimal offline algorithm to Problem (9), respectively. Next, we present an online algorithm ON whose competitive ratio is strictly less than 2, i.e.,

$$\max_{\sigma \in \sigma, \rho \in \rho} \frac{T_{\text{ON}}(\sigma, \rho)}{T_O(\sigma, \rho)} < 2$$

Online Algorithm ON: The algorithm waits till time T_{start} which is the earliest energy arrival at transmitter or time addition at receiver such that using the energy $\mathcal{E}(T_{\text{start}})$ and time $\Gamma(T_{\text{start}})$, B_0 or more bits can be transmitted, i.e.,

$$T_{\text{start}} = \min t \text{ s.t. } \Gamma(t)g\left(\frac{\mathcal{E}(t)}{\Gamma(t)}\right) \geq B_0. \quad (55)$$

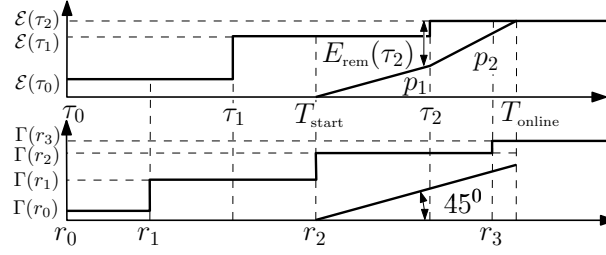


Fig. 9. An example for online algorithm ON.

Starting at T_{start} , ON transmits with power l_1 , such that $\frac{\mathcal{E}(T_{\text{start}})}{l_1}g(l_1) = B_0$. After T_{start} , at every energy arrival epoch τ_j of the transmitter, the transmission power is changed to l_j such that

$$\frac{E_{\text{rem}}(\tau_j)}{l_j}g(l_j) = B_{\text{rem}}(\tau_j). \quad (56)$$

Transmission power is not changed at any ‘time’ arrival r_j at the receiver after T_{start} , because there is sufficient receiver time already available to finish transmission.

Algorithm 5 Online Algorithm ON for energy harvesting transmitter and receiver.

1 **Input:** Bits to transmit B_0 ; \mathcal{E}_i, Γ_i for $\tau_i, r_i \leq t$ where t is the present time instant.

2 $T_{\text{start}} = \min t$ s.t. $\Gamma(t)g\left(\frac{\mathcal{E}(t)}{\Gamma(t)}\right) \geq B_0$

3 $B_{\text{rem}} = B_0, E_{\text{rem}} = \mathcal{E}(T_{\text{start}}), m = T_{\text{start}}$

4 Transmit at power p such that $\frac{E_{\text{rem}}}{p}g(p) = B_{\text{rem}}$

5 **while** $t \leq \left(m + \frac{E_{\text{rem}}}{p}\right)$ **do**

6 **if** $t = \tau_i$ for some i **then**

7 $B_{\text{rem}} = B_{\text{rem}} - (\tau_i - m)g(p)$

8 $E_{\text{rem}} = E_{\text{rem}} + \mathcal{E}_i - (\tau_i - m)p$

9 $m = \tau_i$

10 **end if**

11 Transmit at power p such that $\frac{E_{\text{rem}}}{p}g(p) = B_{\text{rem}}$

12 **end while**

Example: Fig. 9 shows the output of the proposed online algorithm ON, (55) is not satisfied at time τ_0, r_1 , and τ_1 . At time r_2 , (55) is satisfied and transmission starts with a power l_1 such that at rate $g(l_1)$, B_0 bits can be sent in $\mathcal{E}(r_2)/l_1$ time. Transmission power changes to l_2 at time τ_2 such that $\frac{E_{\text{rem}}(\tau_2)}{l_2}g(l_2) = B_{\text{rem}}(\tau_2)$, and so on.

Next, we present certain properties of ON which would help us prove that it is 2-competitive. Lemma 13 proves that similar to the optimal offline algorithm (Lemma 1), ON also has non-decreasing transmission powers.

Lemma 13. *The transmission powers are non-decreasing with time for ON.*

Proof. Combined with proof of Lemma 14. □

Lemma 14 presented below is the key observation to proving Theorem 4. It helps provide a much shorter and elegant proof for competitive ratio less than 2, compared to the proof presented in [9] with no receiver constraints.

Lemma 14. *If power transmitted by ON at time t is l , then $\frac{\mathcal{E}(t)}{l}g(l) \leq B_0, \forall t \in [T_{\text{start}}, T_{\text{ON}}(\sigma, \rho)]$, with equality only at $t = T_{\text{start}}$.*

Proof. After time T_{start} , power of ON is updated at each transmitter energy arrival epoch τ_j . Hence, $b_i = \tau_j$ for some j , and l_i and $\mathcal{E}(t)$ remains constant in $t \in [b_i, b_{i+1})$. Therefore, it is enough to prove that $\frac{g(l_i)}{l_i} \leq \frac{B_0}{\mathcal{E}(b_i)}$ for $i \in \{1, \dots, M\}$. We prove this by induction on $i \in \{1, 2, \dots, M\}$.

With $b_1 = T_{\text{start}}$, the base case follows since at time T_{start} , $\frac{\mathcal{E}(T_{\text{start}})}{l_1}g(l_1) = B_0$. Now, assume $\frac{g(l_{k-1})}{l_{k-1}} \leq \frac{B_0}{\mathcal{E}(b_{k-1})}$ to be true

for $k \in \{2, \dots, M\}$. As $b_k = \tau_j$ for some j ,

$$\begin{aligned} \frac{l_k}{g(l_k)} &= \frac{E_{\text{rem}}(b_k)}{B_{\text{rem}}(b_k)}, \\ &= \frac{E_{\text{rem}}(b_{k-1}) - l_{k-1}(b_k - b_{k-1}) + E_j}{B_{\text{rem}}(b_{k-1}) - g(l_{k-1})(b_k - b_{k-1})}, \\ &\stackrel{(a)}{=} \frac{l_{k-1}}{g(l_{k-1})} + \frac{E_j}{B_{\text{rem}}(b_{k-1})\gamma} \\ &\stackrel{(b)}{>} \frac{\mathcal{E}(b_{k-1})}{B_0} + \frac{E_j}{B_0} \\ &= \frac{\mathcal{E}(b_k)}{B_0}. \end{aligned}$$

where (a) follows from $\frac{B_{\text{rem}}(b_{k-1})}{E_{\text{rem}}(b_{k-1})} = \frac{g(l_{k-1})}{l_{k-1}}$ and defining $\gamma = \left(1 - \frac{l_{k-1}(b_k - b_{k-1})}{E_{\text{rem}}(b_{k-1})}\right) < 1$, (b) uses induction hypothesis for the first term, along with $B_{\text{rem}}(b_{k-1})\gamma < B_0$ for the second term. This completes the proof of Lemma 14. From (a), we can see that $g(l_k)/l_k < g(l_{k-1})/l_{k-1}$. Hence, by monotonicity of $g(p)/p$,

$$l_k > l_{k-1}, \quad \forall k \in \{2, \dots, M\}, \quad (57)$$

proving Lemma 13. \square

Lemma 15 establishes that the start time of ON must be earlier than the finish time of the optimal offline algorithm. Let $T_{\text{start}}(\sigma, \rho)$ be the starting time of ON for input (σ, ρ) .

Lemma 15. *With ON, for any input (σ, ρ) , $T_{\text{start}}(\sigma, \rho) < T_O(\sigma, \rho)$.*

Proof. We prove this Lemma via contradiction. We fix an input (σ, ρ) and show the result. We drop the suffix (σ, ρ) for each of presentation. Suppose $T_{\text{start}} \geq T_O$. From (55), either $T_{\text{start}} = \tau_i$ for some i and/or $T_{\text{start}} = r_j$ for some j . Let $T_{\text{start}} = \tau_i$. Since the optimal offline algorithm $\{\mathbf{p}, \mathbf{s}, N\}$ finishes before T_{start} (which follows from our hypothesis), at the start time of the online algorithm, the maximum (cumulative) energy utilized by the optimal offline algorithm $\{\mathbf{p}, \mathbf{s}, N\}$ is at most the energy arrived till time T_{start}^- . So,

$$\sum_{i: p_i \neq 0} p_i(s_{i+1} - s_i) \leq \mathcal{E}(T_{\text{start}}^-) = \mathcal{E}(T_{\text{start}}) - \mathcal{E}_i \neq \mathcal{E}(T_{\text{start}}). \quad (58)$$

Similarly, if $T_{\text{start}} = r_j$, then the maximum time for which the receiver can be *on* is $\Gamma(T_{\text{start}}^-)$. So,

$$\sum_{i: p_i \neq 0} (s_{i+1} - s_i) \leq \Gamma(T_{\text{start}}^-) = \Gamma(T_{\text{start}}) - \Gamma_j \neq \Gamma(T_{\text{start}}). \quad (59)$$

Therefore, the total number of bits transmitted by the optimal offline algorithm $\{\mathbf{p}, \mathbf{s}, N\}$ is given by

$$\begin{aligned} &\sum_{i=1, p_i \neq 0}^N g(p_i)(s_{i+1} - s_i) \\ &\stackrel{(a)}{\leq} g\left(\frac{\sum_{i: p_i \neq 0} p_i(s_{i+1} - s_i)}{\sum_{j: p_j \neq 0} (s_{j+1} - s_j)}\right) \sum_{j: p_j \neq 0} (s_{j+1} - s_j), \\ &\stackrel{(b)}{\leq} g\left(\frac{\mathcal{E}(T_{\text{start}}^-)}{\Gamma(T_{\text{start}}^-)}\right) \Gamma(T_{\text{start}}^-) \\ &\stackrel{(c)}{<} B_0, \end{aligned} \quad (60)$$

where (a) follows from Jensen's inequality since $g(p)$ is concave, (b) follows from monotonicity of $g(p)/p$ and (58), (59), and (c) follows from (55). From (60), we can conclude that the optimal offline algorithm transmits $\sum_{i=1, p_i \neq 0}^N g(p_i)(s_{i+1} - s_i)$ bits which is less than B_0 , and therefore we arrive at a contradiction. \square

Finally, Theorem 4 proves that ON finishes strictly before twice the time taken by the optimal offline algorithm.

Theorem 4. *The competitive ratio of ON is less than 2.*

Proof. Let ON transmit with power l_k at time T_O^- . Since $T_{\text{start}} < T_O$ by Lemma 15, $l_k > 0$. Let $b_k < T_O$ be the time where transmission starts with power l_k . By definition, $\sum_{i=k}^M g(l_i)(b_{i+1} - b_i) = B_{\text{rem}}(b_k)$. From Lemma 13,

$$(b_{N+1} - b_k) \leq \frac{B_{\text{rem}}(b_k)}{g(l_k)} = \frac{E_{\text{rem}}(b_k)}{l_k} \leq \frac{\mathcal{E}(b_k)}{l_k} \leq \frac{\mathcal{E}(T_O^-)}{l_k}. \quad (61)$$

Applying Lemma 14 at time T_O^- ,

$$\frac{\mathcal{E}(T_O^-)}{l_k} g(l_k) \leq B_0 \stackrel{(a)}{\leq} T_O g\left(\frac{\mathcal{E}(T_O^-)}{T_O}\right), \quad (62)$$

where (a) holds because the maximum number of bits sent by the optimal offline algorithm by time T_O can be bounded by $T_O g\left(\frac{\mathcal{E}(T_O^-)}{T_O}\right)$ due to concavity of $g(p)$. By monotonicity of $g(p)/p$, from (62), it follows that $\frac{\mathcal{E}(T_O^-)}{l_k} \leq T_O$. Combining this with (61), $(b_{N+1} - b_k) \leq T_O$. As $b_k < T_O$, we calculate the competitive ratio as,

$$r = \max_{\sigma \in \Sigma, \rho \in \mathcal{P}} \frac{T_{ON}(\sigma, \rho)}{T_O(\sigma, \rho)} = \frac{(b_{N+1} - b_k) + b_k}{T_O} < 2.$$

□

The next Theorem establishes that ON is an optimal online algorithm by showing that the competitive ratio of any online algorithm is arbitrarily close to 2.

Theorem 5. *ON is an optimal online algorithm.*

Proof. We will construct a set of two energy arrival sequences at the transmitter and the receiver for which the competitive ratio of any online algorithm is arbitrarily close to 2 for at least one of the two sequences.

In order to calculate a lower bound r_ℓ to r (54), we consider $\sigma_s \subseteq \sigma$ and $\rho_s \subseteq \rho$, a small subset of all possible energy harvesting (EH) sequences. Then,

$$r \geq r_\ell = \min_{A \in \mathbf{A}} \max_{\sigma \in \Sigma, \rho \in \mathcal{P}} \frac{T_A(\sigma, \rho)}{T_O(\sigma, \rho)}. \quad (63)$$

The idea behind the proof is to construct a set of two possible EH sequences $\sigma_s = \{\sigma_1, \sigma_2\}$ at the transmitter with the same EH profile $\rho_s = \{\rho_1\}$ at the receiver, where, with σ_1 , the online algorithm ON provides a finish time ratio $\left(\frac{T_{ON}(\sigma_1, \rho_1)}{T_O(\sigma_1, \rho_1)}\right)$ of 1, and with σ_2 it leads to a finish time ratio close to 2. We then proceed to show that the minimum finish time for σ_s, ρ_s over all algorithms in \mathbf{A} is achieved by ON. In doing so, we lower bound r_ℓ by a value arbitrarily close to 2. Combining this with the fact that $r < 2$ for ON (from Theorem 4), we can say that ON achieves the optimal competitive ratio. Now, it remains to show that $r_\ell \geq 2^-$.

Next, we explain the construction of σ_s . Let σ_1 consist of only one EH arrival \mathcal{E}_0 at time $\tau_0 = 0$, and let σ_2 represent the EH sequence $\{\mathcal{E}_0, \mathcal{E}_1\}$ occurring at time $\tau_0 = 0$ and $\tau_1 = 1$. We assume that the receiver has only one ‘time’ arrival of $\Gamma_0 = T$ at time $t = 0$, i.e. $\rho = \{T\}$ at time $r_0 = 0$. Let \mathcal{E}_0 and $T \gg 0$ be chosen such that $B_0 = Tg\left(\frac{\mathcal{E}_0}{T}\right)$. Let \mathcal{E}_1 be such that $B_0 = \tau_1 g\left(\frac{\mathcal{E}_0 + \mathcal{E}_1}{\tau_1}\right)$. The performance of algorithm OFFM and the online algorithm ON for energy arrival sequences $\{\sigma_1, \sigma_2\}$ is depicted in Fig. 10. Clearly, both OFFM and ON follow a constant power transmission policy for σ_1 where power $\frac{\mathcal{E}_0}{T}$ is transmitted from time 0 to T . For σ_2 , OFFM transmits with power $\frac{\mathcal{E}_0}{\tau_1}$ from time 0 to τ_1 , and power $\frac{\mathcal{E}_1}{T_1 - \tau_1}$ from τ_1 to T_1 , where T_1 is calculated by

$$(T_1 - \tau_1)g\left(\frac{\mathcal{E}_1}{T_1 - \tau_1}\right) = B_0 - \tau_1 g\left(\frac{\mathcal{E}_0}{\tau_1}\right). \quad (64)$$

Compared to this, with σ_2 , ON transmits with power $\frac{\mathcal{E}_0}{T}$ from time 0 to τ_1 and power $\frac{\mathcal{E}_1 + \mathcal{E}_0(1 - \tau_1/T)}{T_2 - \tau_1}$ from time τ_1 to T_2 , where T_2 is given by

$$(T_2 - \tau_1)g\left(\frac{\mathcal{E}_1 + \mathcal{E}_0(1 - \tau_1/T)}{T_2 - \tau_1}\right) = B_0 - \tau_1 g\left(\frac{\mathcal{E}_0}{T}\right). \quad (65)$$

Therefore,

$$\frac{T_{ON}(\sigma_1, \rho_1)}{T_O(\sigma_1, \rho_1)} = 1, \text{ while } \frac{T_{ON}(\sigma_2, \rho_1)}{T_O(\sigma_2, \rho_1)} = \frac{T_2}{T_1}. \quad (66)$$

Now consider any online algorithm $A \in \mathbf{A}$. Since, A is assumed to use only causal information regarding energy harvests, it would generate identical power transmission profile with EH sequence σ_1 and σ_2 for time 0 to τ_1 . Let A use α fraction of energy \mathcal{E}_0 till time τ_1 . Hence, we can characterize every online algorithm A by α , the fraction of energy it uses in time $[0, \tau_1]$, and

$$r_\ell \geq \min_{\alpha \in [0, 1]} \max_{\sigma \in \{\sigma_1, \sigma_2\}} \frac{T_A(\sigma, \rho_1)}{T_O(\sigma, \rho_1)}. \quad (67)$$

Let us denote the corresponding value of α for algorithm ON as $\alpha' = 1 - \tau_1/T$. The total receiver time being $\Gamma_0 = T$, the maximum number of bits that can be transmitted by any online algorithm A with particular choice of α , for EH sequence σ_1 is given by,

$$B_\alpha = \tau_1 g\left(\frac{\alpha \mathcal{E}_0}{\tau_1}\right) + (T - \tau_1)g\left(\frac{(1 - \alpha)\mathcal{E}_0 + \mathcal{E}_1}{T - \tau_1}\right). \quad (68)$$

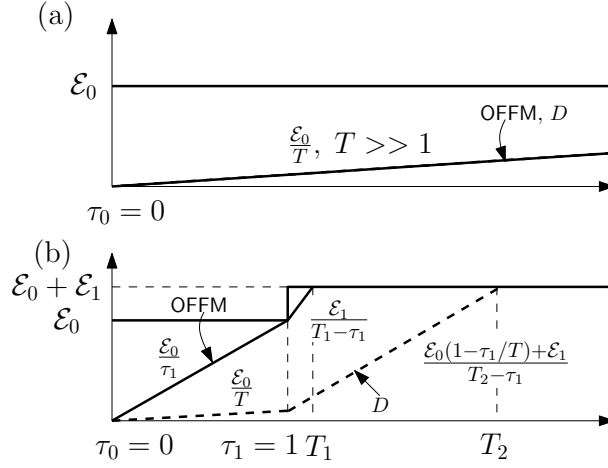


Fig. 10. Transmission policy of OFFM and the online algorithm ON for EH profile (a) σ_1 and (b) σ_2 .

Because of the concavity of rate function g , from (64), we can see that $B_\alpha \leq B_0$, with equality iff $\alpha = \alpha'$. So, for $\alpha \neq \alpha'$, algorithm A cannot transmit B_0 bits with EH sequence σ_1 . Therefore, in the RHS (67), we only concern ourselves with the performance of online algorithm with $\alpha = \alpha'$, i.e. ON. Hence,

$$r_\ell \geq \max \left(\frac{T_{\text{ON}}(\sigma_1, \rho_1)}{T_O(\sigma_1, \rho_1)}, \frac{T_{\text{ON}}(\sigma_2, \rho_1)}{T_O(\sigma_2, \rho_1)} \right) \quad (69)$$

$$= \frac{T_{\text{ON}}(\sigma_2, \rho_1)}{T_O(\sigma_2, \rho_1)} = \frac{T_2}{T_1}, \quad (70)$$

where the last equality follows from (66).

Therefore, we only need to show that $\frac{T_2}{T_1} \geq 2^-$ for ON by choosing parameters \mathcal{E}_0 and T . It is difficult to obtain a closed form expression for $\frac{T_2}{T_1}$ in terms of relevant parameters, hence we lower bound $\frac{T_2}{T_1}$ by constructing an example sequence $\{\sigma_1, \sigma_2, \rho_1\}$ as follows. With $\mathcal{E}_0 = 10^{-4}$, $T = 10^4$, and $g(p) = 0.5 \log_2(1 + p)$, we get $\frac{T_2}{T_1} = 2 - 2.49 \times 10^{-4}$. Similarly, by increasing T and decreasing \mathcal{E}_0 towards 0, we can keep pushing $\frac{T_2}{T_1}$ arbitrarily close to 2. This completes the proof. \square

Discussion: In this section, we derived an optimal online algorithm when EH is employed at both the transmitter and the receiver. First, we proposed an online algorithm and showed that it finishes the transmission of required number of bits in at most twice the time an optimal offline algorithm takes knowing all energy arrivals non-causally. Moreover, the online algorithm is independent of the energy arrival distributions both at the transmitter and the receiver, so has built-in robustness. Also, note that the proof of Theorem 4 does not explicitly require to know the exact structure of the optimal offline algorithm. Thereafter, to complete the characterization of optimal online algorithms, we showed that no online algorithm can do better than the proposed online algorithm by constructing a set of energy arrival sequences for which any online algorithm will have competitive ratio arbitrarily close to two for at least one of the energy arrival sequences. Typically, finding a (tight) lower bound on the competitive ratio for all online algorithms is a hard problem, but we are able to accomplish this for the transmission finish time minimization problem.

After examining the case of EH being employed at both the transmitter and the receiver with no battery constraint until now in this paper, we next consider the more reasonable model of a finite battery availability at both the transmitter and the receiver, and derive online algorithm with bounded competitive ratio.

VII. ONLINE ALGORITHMS WITH FINITE BATTERY AT TRANSMITTER AND RECEIVER

In previous sections, an infinite battery capacity was assumed at both the transmitter and the receiver. In this section, to make the discussion more practical, we consider the case when both the transmitter and receiver battery have finite capacity. Also, we consider the online setting for obvious practical reasons.

A. EH only at the transmitter

For ease of exposition, we first discuss the finite battery model where only the transmitter is EH powered, while the receiver is powered by a conventional power source. We extend the analysis to include an EH powered receiver in Section VII-B. With finite battery capacity, similar to Section VI, under a worst case input for energy harvests, the online algorithm might not finish transmission of B_0 bits ever, while an offline algorithm can, making the competitive ratio infinity. Thus, we consider the non-degenerate online setting, where the amount of energy arriving at any instant is a random variable whose probability

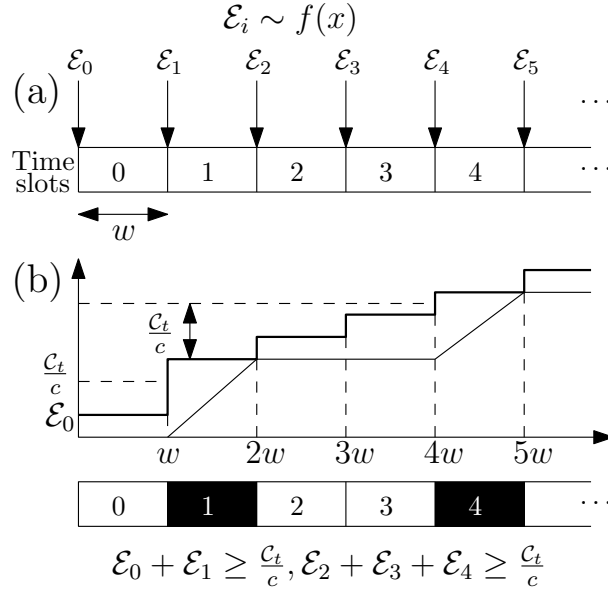


Fig. 11. (a) Transmitter model for slotted energy arrival (b) An example for Accumulate&Dump algorithm.

density function (PDF) $f(x)$ is known ahead of time. Note that on the realization basis, only causal information is revealed to any online algorithm.

For simplicity, we divide time into slots of length w , with \mathcal{E}_i amount of energy arriving in the i^{th} slot. We also assume that energy is harvested at the beginning of the slot. The amount of energy arriving in any slot i , \mathcal{E}_i , is assumed to follow an i.i.d. PDF $f(x)$ for all $i \geq 0$. The transmitter is assumed to have a battery capacity \mathcal{C}_t . Thus without loss of generality we assume that $f(x) = 0$ for $x > \mathcal{C}_t$. Following Problem (14), we want to transmit B_0 bits in total in minimum time under this online setting with finite battery capacity at the transmitter. The system model is shown in Fig. 11 (a). With randomized energy inputs, we consider the expected competitive ratio as the performance metric to design online algorithms, that is defined as the expectation of the ratio of the time taken by an online algorithm and the time taken by an optimal offline algorithm. We next present an online algorithm which we call Accumulate&Dump to upper bound the expected competitive ratio.

Algorithm Accumulate&Dump: In the first iteration, algorithm Accumulate&Dump waits for \mathcal{N} slots such that at least \mathcal{C}_t/c amount of battery capacity is filled, where $c \geq 1$ is a positive constant in the algorithm. The value c is dependent on $f(x)$ and we will calculate the best choice of c for a given distribution $f(x)$ while analysing the algorithm. Clearly \mathcal{N} is a random variable given by,

$$\mathcal{N} = \min \left\{ n : \sum_{i=0}^{n-1} \mathcal{E}_i \geq \frac{\mathcal{C}_t}{c} \right\}. \quad (71)$$

After accumulating at least \mathcal{C}_t/c amount of energy, Accumulate&Dump uses all of the available energy $\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i$ in the battery with a constant rate in w amount of time i.e. within the \mathcal{N}^{th} slot. With an empty battery at the end of \mathcal{N}^{th} slot, the transmitter starts accumulating energy afresh and continues the above process until it transmits all B_0 amount of bits.

Example: Fig.11 (b) shows an example of running Accumulate&Dump with $\mathcal{N} = 2$ in the first iteration and $\mathcal{N} = 3$ in the next.

Analysis: Consider the sum process of the i.i.d. random variables \mathcal{E}_i , $\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i$, where \mathcal{N} defined in (71). Let us denote condition H as

$$H \equiv \left(\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i \geq \mathcal{C}_t/c \text{ and } \sum_{i=0}^{\mathcal{N}-2} \mathcal{E}_i < \mathcal{C}_t/c \right). \quad (72)$$

Note that the stopping condition in (71) is equivalent to H , as $f(x) = 0$ for $x < 0$. Lemma 17 formulates an expression for the expected value of \mathcal{N} . First, we state the form of Walds' equation [16] that we use in Lemma 16.

Lemma 16. [Walds equation] If \mathcal{S} is a stopping time with respect to an i.i.d. sequence $\{X_n : n \geq 1\}$, and if $\mathbf{E}[\mathcal{S}] < \infty$ and $\mathbf{E}[|X_i|] < \infty$, then $\mathbf{E}[\sum_{i=1}^{\mathcal{S}} X_i] = \mathbf{E}[\mathcal{S}]\mathbf{E}[X_i]$.

Lemma 17. $\mathbf{E}[\mathcal{N}] = \frac{\mathbf{E}[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i]}{\mathbf{E}[\mathcal{E}_0]}.$

Proof. Clearly, \mathcal{N} is a stopping time. The proof directly follows from Lemma 16, once we show that $\mathbf{E}[\mathcal{N}]$ is finite that is proved in Appendix A. \square

Next, we analyze the competitive ratio of the online algorithm Accumulate&Dump. Since, \mathcal{E}_i 's are random variables, we use expected competitive ratio analysis for Accumulate&Dump, and prove an upper bound to it in Theorem 6. In doing so, we primarily consider a class of distributions that satisfy the following condition.

Assumption 1. $\mathbf{E}[\mathcal{E}_0 | \mathcal{E}_0 \geq \gamma] \leq \gamma + \mathbf{E}[\mathcal{E}_0], \forall \gamma \in [0, C_t]$.

This assumption simply means that the expected jump size given that it is larger than γ is no more than if the origin is shifted to γ and the process takes an i.i.d. jump from there. Note that most light-tailed continuous distributions satisfy Assumption 1. For example, uniform distribution satisfies Assumption 1 with a strict inequality, while the exponential distribution satisfies Assumption 1 with an equality.

Remark 1. To find a bound on $\mathbf{E}[\mathcal{N}]$, we need an upper bound on $\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i\right]$. Towards that end, we need a bound on the expected value of the \mathcal{N}^{th} increment of process \mathcal{E}_i given that $\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i > \frac{C_t}{c}$. Without Assumption 1, knowing that $\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i > C_t/c$, there is no easy way of bounding the value $\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i\right]$ except of course the trivial bound of C_t . For example, suppose \mathcal{E}_i 's have a Bernoulli distribution over $\{0, x\}$ with probability $\{p, 1-p\}$. If x is large, and we condition on $x > 0$, then $\mathbf{E}[x] = x$. Heavy tailed distributions also do not allow any bound on the expected value of $\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}} \mathcal{E}_i\right]$ at the cross-over point. As we will see in Proof of Theorem 6, Assumption 1 is sufficient to obtain non-trivial bound on the expected jump size given that the jump is larger than a certain threshold.

Theorem 6. The expected competitive ratio of Accumulate&Dump algorithm is finite.

Proof. Let the number of slots taken by the optimal offline algorithm to finish transmitting B_0 bits is \mathcal{S}_{off} and the number of slots taken by Accumulate&Dump to complete is \mathcal{S}_{on} . Since the maximum amount of energy harvested in one slot is bounded by C_t , to bound \mathcal{S}_{off} , we consider the best case scenario (that is the fastest completion of transmission) where C_t amount of energy arrives in each slot. In this best case, the number of bits transmitted per slot, i.e. slot width of w , is $wg(C_t/w)$, and the total number of slots taken to transmit B_0 bits is $\left\lceil \frac{B_0}{wg(C_t/w)} \right\rceil$. Therefore,

$$\mathcal{S}_{\text{off}} \geq \frac{B_0}{wg(C_t/w)}. \quad (73)$$

Now, we can write the expected competitive ratio as,

$$\mathbf{E}[r] = \mathbf{E}\left[\frac{\mathcal{S}_{\text{on}}}{\mathcal{S}_{\text{off}}}\right] \leq \mathbf{E}\left[\frac{\mathcal{S}_{\text{on}}}{\frac{B_0}{wg(C_t/w)}}\right] = \frac{\mathbf{E}[\mathcal{S}_{\text{on}}]}{\frac{B_0}{wg(C_t/w)}}. \quad (74)$$

In each iteration, Accumulate&Dump waits for $(\mathcal{N} - 1)$ slots by which time it accumulates at least C_t/c amount of energy, and then uses all the accumulated energy in the \mathcal{N}^{th} slot for transmission. Hence, at least $wg\left(\frac{C_t}{cw}\right)$ bits are transmitted in the \mathcal{N}^{th} slot by Accumulate&Dump, where \mathcal{N} is defined in (71). Note that \mathcal{N} is i.i.d. random variable over all iterations of Accumulate&Dump. Thus the number of bits transmitted by Accumulate&Dump in time $\mathcal{N}w$ is $wg\left(\frac{C_t}{cw}\right)$. This implies that

the maximum number of iterations (say m) taken by Accumulate&Dump to transmit B_0 bits is $\left\lceil \frac{B_0}{wg\left(\frac{C_t}{cw}\right)} \right\rceil$. Hence,

$$\mathbf{E}[\mathcal{S}_{\text{on}}] = \mathbf{E}[\mathcal{N} \times m] \quad (75)$$

$$\leq \mathbf{E}[\mathcal{N}] \left\lceil \frac{B_0}{wg\left(\frac{C_t}{cw}\right)} \right\rceil \stackrel{(a)}{\approx} \mathbf{E}[\mathcal{N}] \frac{B_0}{wg\left(\frac{C_t}{cw}\right)}, \quad (76)$$

where (a) follows under the assumption that $m \gg 1$.

Under Assumption 1, as shown in Appendix B,

$$\mathbf{E}[\mathcal{N}] \leq \frac{C_t/c}{\mathbf{E}[\mathcal{E}_0]} + 1. \quad (77)$$

Without assumption 1, using the trivial upper bound $\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i\right] \leq C_t$, we get

$$\mathbf{E}[\mathcal{N}] \leq \frac{C_t/c + C_t}{\mathbf{E}[\mathcal{E}_0]} \quad (78)$$

for any general distribution $f(x)$, which is also shown in Appendix B.

Thus, from (74), (76), and (77), we can write the competitive ratio under Assumption 1 as,

$$\begin{aligned}\mathbf{E}[r] &\leq \frac{\left(\frac{C_t/c}{\mathbf{E}[\mathcal{E}_0]} + 1\right) \frac{B_0}{wg\left(\frac{C_t}{cw}\right)}}{\frac{B_0}{wg(C_t/w)}}, \\ &= \left(\frac{C_t/c}{\mathbf{E}[\mathcal{E}_0]} + 1\right) \frac{g\left(\frac{C_t}{w}\right)}{g\left(\frac{C_t}{cw}\right)}.\end{aligned}\quad (79)$$

Recall that we can choose the parameter c . For $c = \frac{C_t}{\mathbf{E}[\mathcal{E}_0]}$, (79) reduces to $\mathbf{E}[r] \leq \frac{2g\left(\frac{C_t}{w}\right)}{g\left(\frac{\mathbf{E}[\mathcal{E}_0]}{w}\right)}$. With $g(p) = 0.5 \log_2(1+p)$,

$\frac{g\left(\frac{C_t}{w}\right)}{g\left(\frac{\mathbf{E}[\mathcal{E}_0]}{w}\right)}$ is constant for any distribution $f(x)$, where C_t scales polynomially with $\mathbf{E}[\mathcal{E}_0]$. Thus, we get a constant upper bound for $\mathbf{E}[r]$ under Assumption 1.

For any general distribution $f(x)$, from (78),

$$\mathbf{E}[r] \leq \left(\frac{C_t/c + C_t}{\mathbf{E}[\mathcal{E}_0]}\right) \frac{g\left(\frac{C_t}{w}\right)}{g\left(\frac{C_t}{cw}\right)},$$

which can shown to be finite for $c = \frac{C_t}{\mathbf{E}[\mathcal{E}_0]}$, as above, but now the bound depends on system parameters C_t and $\mathbf{E}[\mathcal{E}_0]$. \square

Next, we evaluate the derived bounds for particular energy arrival distributions.

Example 1. For uniform distribution $f(x) = 1/C_t, 0 \leq x \leq C_t$, we have $\mathbf{E}[\mathcal{E}_0] = C_t/2$ and we can reduce (79) to $\mathbf{E}[r] \leq 2 \frac{\log_2\left(1 + \frac{C_t}{w}\right)}{\log_2\left(1 + \frac{C_t}{2w}\right)} < 4$.

Example 2. For exponential energy arrival at the transmitter, we can write the probability density function $f(x)$ as,

$$f(x) = \lambda e^{-\lambda x}, \quad 0 \leq x < C_t, \quad (80)$$

$$= e^{-\lambda C_t}, \quad x = C_t, \quad (81)$$

$$= 0, \quad x > C_t, x < 0. \quad (82)$$

Let us assume that the probability of the energy arrival being more than the battery capacity is given by $10^{-\epsilon} = f(C_t)$ for $\epsilon > 0$. Note that $\mathbf{E}[\mathcal{E}_0] = (1 - 10^{-\epsilon})/\lambda$. So, with $c = \frac{C_t}{\mathbf{E}[\mathcal{E}_0]}$, the upper bound (79) reduces to

$$2 \frac{\log_2\left(1 + \frac{\epsilon \ln 10}{\lambda w}\right)}{\log_2\left(1 + \frac{(1-10^{-\epsilon})}{\lambda w}\right)} < 2 \max\left(1, \frac{\epsilon \ln 10}{1 - 10^{-\epsilon}}\right) \quad (83)$$

$$\approx 2, \quad \text{for } \epsilon < 0.43 \quad (84)$$

$$\approx 4.6\epsilon, \quad \text{for } \epsilon \geq 0.43. \quad (85)$$

B. EH at both transmitter and receiver:

After analyzing the expected competitive ratio when only the transmitter is powered by EH, in this subsection, we generalize expected competitive ratio analysis of subsection VII-A to allow for both transmitter and receiver to be powered by EH and where both have finite battery capacities. The transmitter model remains as defined in subsection VII-A, while for the receiver we assume that \mathcal{R}_i , the energy arriving at each slot, is i.i.d. with PDF $h(x)$. The receiver has a finite battery capacity C_r and it uses P_r amount of power to be *on*. Also, $h(x) = 0$ for $x > C_r$ and $x < 0$.

In this model, we propose a natural extension of Accumulate&Dump as follows, *Algorithm modified Accumulate&Dump*: In the first iteration, the algorithm waits for \mathcal{N} energy arrivals such that at least C_t/c amount of energy is harvested at the transmitter, and at least $P_r w$ amount is accumulated at the receiver, where $c \geq 1$. That is,

$$\mathcal{N} = \min \left\{ n : \sum_{i=0}^{n-1} \mathcal{E}_i \geq \frac{C_t}{c} \text{ and } \sum_{i=0}^{n-1} \mathcal{R}_i \geq P_r w \right\}. \quad (86)$$

In the \mathcal{N}^{th} slot, the transmitter uses all the accumulated energy to transmit at a constant rate, and the receiver is also *on*. After this, the system is essentially reset and the algorithm proceeds to the next iteration.

For this modified Accumulate&Dump algorithm we provide a expected competitive ratio bound in Theorem 7.

Theorem 7. *The expected competitive ratio when both transmitter and receiver are powered by EH is upper bounded by*

$$\mathbf{E}[r] \leq \left(\frac{C_r + P_r w}{\mathbf{E}[\mathcal{R}_0]} + \frac{C_t + C_t/c}{\mathbf{E}[\mathcal{E}_0]} \right) \frac{g(\frac{C_t}{w})}{g(\frac{C_t}{cw})},$$

for any general distribution of $f(x)$ and $h(x)$, and

$$\mathbf{E}[r] \leq \left(\frac{P_r w}{\mathbf{E}[\mathcal{R}_0]} + \frac{C_t/c}{\mathbf{E}[\mathcal{E}_0]} + 2 \right) \frac{g(\frac{C_t}{w})}{g(\frac{C_t}{cw})},$$

when $f(x)$ and $h(x)$ satisfy Assumption 1.

Proof. Let us define \mathcal{N}' and \mathcal{N}'' as

$$\mathcal{N}' = \min \left\{ n : \sum_{i=0}^{n-1} \mathcal{E}_i \geq \frac{C_t}{c} \right\},$$

$$\mathcal{N}'' = \min \left\{ n : \sum_{i=0}^{n-1} \mathcal{R}_i \geq P_r w \right\}.$$

Hence, from (86), $\mathcal{N} = \max(\mathcal{N}', \mathcal{N}'')$. Using $\mathbf{E}[\mathcal{N}] < \mathbf{E}[\mathcal{N}'] + \mathbf{E}[\mathcal{N}'']$, the rest of the proof follows along similar lines as the proof of Theorem 6. \square

Discussion: In this section, we proposed a simple online algorithm that accumulates energy upto a certain threshold and transmits (dumps) all of it as soon as it crosses the threshold. The chosen threshold controls the rate at which algorithm transmits power, larger the threshold less slots are active but with more power and vice versa. The idea behind this algorithm is that given that the rate function is concave (e.g., log), the effect of not transmitting power in every slot is not too large, and one can tradeoff the threshold appropriately to find the best threshold given the energy arrival distribution information. We show that for most ‘nice’ energy arrival distributions that do not have arbitrarily large jumps, we can bound the expected competitive ratio by a constant that does not depend on the system parameters, thus showing that the proposed algorithm is close to optimal and has reasonable performance.

VIII. SIMULATION RESULTS

In this section, we first present a sample run of algorithm OFFM with $B_0 = 1$ as shown in Fig. 12 for a given energy arrival sequence at the transmitter and receiver. We can see from the transmitter-receiver energy profiles that the finish time decreases through policy X_1 to X_4 . At the same, the transmission time increases from policy X_1 to X_4 . Among policies X_i , $i = 1, 2, 3, 4$, X_4 is the first policy whose start time is less than O_{i+1} (not shown in Fig. 12). Hence, by Lemma 12, X_4 is optimal.

Next, we perform simulations to illustrate the competitive ratio performance of the online algorithm from section VI with no battery capacity constraints. The amount of energy harvested at the transmitter, and the energy (or time) harvested at the receiver are drawn from a uniform distribution in $[0, 1]$. The inter-arrival distribution of energy harvests at transmitter and receiver is uniform in $[0, 1]$. The rate function is assumed to be $g(p) = 0.5 \log_2(1 + p)$. Comparison between the online algorithm and OFFM is shown in Fig. 13. We can observe that the competitive ratio is close to 1.5 for different values of B_0 bits, which is far better than the worst case bound of 2 as derived in Theorem 4.

For the finite battery setting, in Fig. 14 (a), we first simulate the case when only the transmitter is powered by EH, where energy arrivals follow an exponential distribution, and demonstrate the competitive ratio of Accumulate&Dump compared to the optimal offline algorithm [13]. In this experiment, we assume the rate function to be given by $g(p) = 0.5 \log_2(1 + p)$, the battery capacity to be 115 units, the slot width to be 5 units, and the energy arrival distribution to be exponential with mean 25. As described in Example 2, the distribution is truncated, i.e. any energy arrival of amount more than C_t is assumed to have a value of exactly C_t . We have chosen our battery capacity so that the value for ϵ comes out to be 2. That is, there is a probability of 0.01 that the energy harvested is more than C_t . Minimizing the upper bound on the expected competitive ratio given in (79), the optimal value comes out to be 3.56 for $c = 5.07$. Although the theoretical upper bound computed is 3.56, we can see that the simulated competitive ratio converges around 1.27.

Then we consider the case when both transmitter and receiver are powered with EH in the finite battery setting, and simulate the competitive ratio in Fig. 14 (b). In this model, both the transmitter and the receiver are assumed to harvest energy from exponential distribution with mean 25, and both have a battery capacity of 115. The receiver *on* power is assumed to be $P_r = 7$. With $w = 5$ and $c = 5.07$, we can see that the upper bound on expected competitive ratio calculated using Theorem 7 turns out to be 8. An important point to note here is that the optimal offline algorithm is not known for the finite battery setting when both the transmitter and the receiver are powered by EH. Thus, to compute the competitive ratio, we consider the optimal offline algorithm [13] when only the transmitter is powered by EH with finite battery, which clearly is

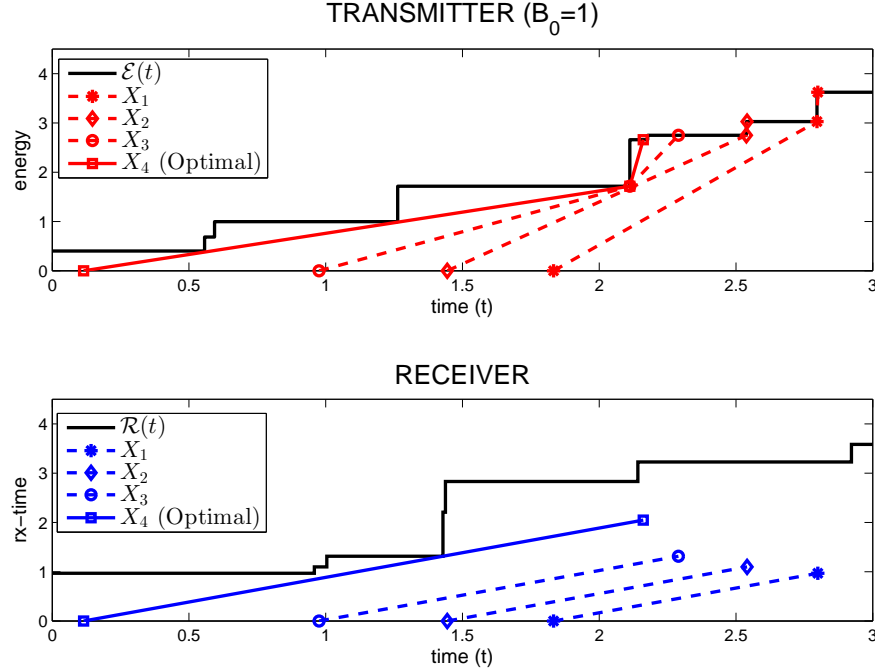


Fig. 12. An example for OFFM algorithm.

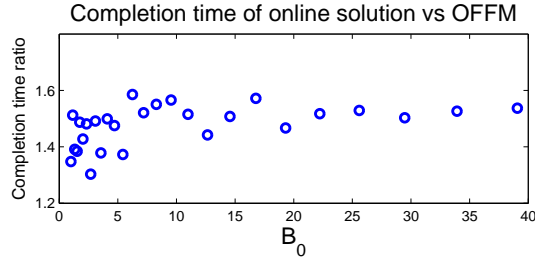


Fig. 13. OFFM algorithm vs online algorithm with no battery capacity constraints.

an upper bound on the performance when both transmitter and receiver are powered by EH. In simulations, we compare the modified Accumulate&Dump algorithm from section VII-B with the optimal offline algorithm presented in [13]. The simulated competitive ratio converges around 1.65.

IX. CONCLUSIONS

In this paper, we have made significant progress in finding optimal transmission strategies when EH is employed at both the transmitter and the receiver. As is evident, EH at both ends is fundamentally different than the case when only the transmitter is powered by EH. With EH at both ends, we have not only found an optimal offline algorithm, which has been accomplished for many other similar but simpler models in past, but also proposed “good” online algorithms for both finite and infinite battery capacities that have provably efficient competitive ratio compared to the offline algorithms. In particular, in the infinite battery case, the proposed online algorithm is also shown to be optimal. One limitation of tx-rx EH model that we glossed over is if there is no centralized controller, how to make transmitter and receiver aware of each others’ battery states. This is actually a fundamental issue, and it would require more sophisticated techniques to solve this more general problem. Some limited results are available in [10].

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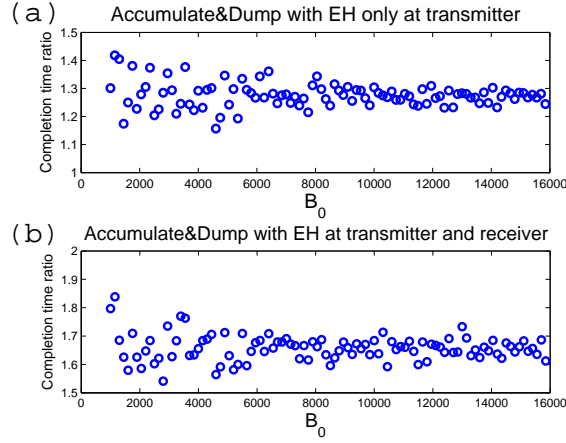


Fig. 14. Comparison of algorithm Accumulate&Dump with the optimal offline algorithm presented in [13] with finite battery capacity in the (a) transmitter model and (b) transmitter-receiver model.

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APPENDIX A PROOF OF LEMMA 17

We seek to apply Wald's equation from Lemma 16, for which we have to prove that $\mathbf{E}[N]$ and $\mathbf{E}[\mathcal{E}_0]$ are finite. $\mathbf{E}[\mathcal{E}_0] < \infty$ follows from the fact that $f(x) = 0$ for $x > \mathcal{C}_t$. We now proceed to prove that $\mathbf{E}[N] < \infty$.

$$\mathbf{E}[N] = \sum_{n=1}^{\infty} P[N > n] \leq \sum_{n=1}^{\infty} P\left[\sum_{i=0}^{n-1} \mathcal{E}_i \leq \frac{\mathcal{C}_t}{c}\right]. \quad (\text{A.1})$$

Let us choose a constant $x \in (0, \mathcal{C}_t]$ such that $P[\mathcal{E}_i > x] > 0$ and say $q = P[\mathcal{E}_i > x]$. Define $Y_i = \mathbb{1}_{\mathcal{E}_i > x}$. Clearly all Y_i 's are i.i.d random variables. Let $k = \lceil \frac{\mathcal{C}_t}{cx} \rceil \Rightarrow kx > \frac{\mathcal{C}_t}{c}$. Now, for any $n > 0$,

$$\sum_{i=0}^{n-1} \mathcal{E}_i \leq \frac{\mathcal{C}_t}{c} \Rightarrow \sum_{i=0}^{n-1} Y_i \leq k, \quad (\text{A.2})$$

$$\begin{aligned} P\left[\sum_{i=0}^{n-1} \mathcal{E}_i \leq \frac{\mathcal{C}_t}{c}\right] &\leq P\left[\sum_{i=0}^{n-1} Y_i \leq k\right], \\ &= \sum_{r=0}^k \binom{n}{r} q^r (1-q)^{n-r}. \end{aligned} \quad (\text{A.3})$$

From (A.1) and (A.3),

$$\mathbf{E}[N] \leq \sum_{n=1}^{\infty} \sum_{r=0}^k \binom{n}{r} q^r (1-q)^{n-r}, \quad (\text{A.4})$$

$$\stackrel{(a)}{\leq} \sum_{n=1}^{\infty} q'^n \sum_{r=0}^k \binom{n}{r}, \quad (\text{A.5})$$

$$\stackrel{(b)}{\leq} \alpha \sum_{n=1}^{\infty} q'^n n^{k+1} \stackrel{(c)}{<} \infty. \quad (\text{A.6})$$

where $q' = \min(q, 1-q)$ in (a). As $\sum_{r=0}^k \binom{n}{r}$ is a polynomial in n with degree $k+1$, (b) follows with some constant α . (c) follows since sequence $q'^n n^{k+1}$ converges in n , which can be easily verified with the ratio test.

Therefore, with $\mathbf{E}[\mathcal{E}_0] < \infty$ and $\mathbf{E}[N] < \infty$, we use Wald's equation to write,

$$\mathbf{E}[N]\mathbf{E}[\mathcal{E}_0] = \mathbf{E}\left[\sum_{i=0}^{N-1} \mathcal{E}_i\right], \quad (\text{A.7})$$

under stopping condition H defined in (72).

APPENDIX B

From Lemma 17, $\mathbf{E}[\mathcal{N}]$

$$\begin{aligned} &= \frac{\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &= \frac{\mathbf{E}\left[\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i | H\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &\stackrel{(a)}{=} \frac{\mathbf{E}\left[\mathbf{E}\left[\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i \middle| \mathcal{E}_{\mathcal{N}-1} \geq \mathcal{C}_t/c - k, \sum_{i=0}^{\mathcal{N}-2} \mathcal{E}_i = k\right]\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &= \frac{\mathbf{E}\left[\mathbf{E}\left[\mathbf{E}\left[k + \mathcal{E}_{\mathcal{N}-1} \middle| \mathcal{E}_{\mathcal{N}-1} \geq \mathcal{C}_t/c - k, \sum_{i=0}^{\mathcal{N}-2} \mathcal{E}_i = k\right]\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &= \frac{\mathbf{E}\left[\mathbf{E}\left[\mathbf{E}\left[k + \mathcal{E}_{\mathcal{N}-1} \middle| \mathcal{E}_{\mathcal{N}-1} \geq \mathcal{C}_t/c - k\right]\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &\stackrel{(b)}{\leq} \frac{\mathbf{E}\left[\mathbf{E}\left[k + \mathcal{C}_t/c - k + \mathbf{E}[\mathcal{E}_{\mathcal{N}-1}]\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &= \frac{\mathbf{E}\left[\mathbf{E}\left[\mathcal{C}_t/c + \mathbf{E}[\mathcal{E}_0]\right]\right]}{\mathbf{E}[\mathcal{E}_0]}, \\ &= \frac{\mathcal{C}_t/c}{\mathbf{E}[\mathcal{E}_0]} + 1, \end{aligned} \quad (\text{B.1})$$

where k is a constant in (a) with $0 \leq k < \mathcal{C}_t/c$ and (b) follows under Assumption 1. For general energy arrival distributions, we can write $\mathbf{E}[\mathcal{N}]$ as,

$$\begin{aligned}
\mathbf{E}[\mathcal{N}] &= \frac{\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-1} \mathcal{E}_i\right]}{\mathbf{E}[\mathcal{E}_0]}, \\
&= \frac{\mathbf{E}\left[\sum_{i=0}^{\mathcal{N}-2} \mathcal{E}_i\right]}{\mathbf{E}[\mathcal{E}_0]} + \frac{\mathbf{E}[\mathcal{E}_{\mathcal{N}-1}]}{\mathbf{E}[\mathcal{E}_0]}, \\
&\stackrel{(a)}{\leq} \frac{\mathcal{C}_t/c}{\mathbf{E}[\mathcal{E}_0]} + \frac{\mathbf{E}[\mathcal{E}_{\mathcal{N}-1}]}{\mathbf{E}[\mathcal{E}_0]}, \\
&\stackrel{(b)}{\leq} \frac{\mathcal{C}_t/c}{\mathbf{E}[\mathcal{E}_0]} + \frac{\mathcal{C}_t}{\mathbf{E}[\mathcal{E}_0]}, \tag{B.2}
\end{aligned}$$

where (a) follows under stopping condition H defined in (72) and (b) follows since, $f(x) = 0$ for $x > \mathcal{C}_t$.